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**MANAGEMENT CONTROL:  
SAFEGUARDING ASSETS AT MULTIPLE LOCATIONS**

**Dissertation**

**Presented in Partial Fulfillment of the Requirements for the  
Degree Doctor of Philosophy in the Graduate School of  
The Ohio State University**

**By**

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**\* \* \* \* \***

**The Ohio State University**

**1995**

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Finally, and most importantly, I thank my wife, Ellie. She has repeatedly made sacrifices in order to encourage me to pursue a Ph.D. Without her, I could not have completed this project.

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## LIST OF SYMBOLS

### PARAMETERS

$V_j$  = asset value at location  $j$

$C_j$  = incremental verification cost at location  $j$

$\alpha_{Hj}$  = probability of detecting a theft at location  $j$ , given the effectiveness of internal controls is high

$\alpha_{Lj}$  = probability of detecting a theft at location  $j$ , given the effectiveness of internal controls is low

$q$  = probability that the effectiveness of internal controls is high

### CHOICE VARIABLES - NON-SEQUENTIAL VERIFICATION (CHAPTER III)

$T_j$  = probability of theft at location  $j$

$A_j$  = probability of verification at location  $j$

### CHOICE VARIABLES - SEQUENTIAL VERIFICATION (CHAPTERS IV AND V)

$T_a$  = probability of theft at location  $a$

$T_b$  = probability of theft at location  $b$

$[A_1]$  = probability of verification at location 1

$[A_2 | A_1D_1]$  = probability of verification at location 2, given verification and detection at location 1

$[A_2 | A_1ND_1]$  = probability of verification at location 2, given verification and no detection at location 1

$[A_2 | NA_1]$  = probability of verification at location 2, given no verification at location 1

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## **CHAPTER I**

### **Introduction**

In any organization safeguarding assets is a primary concern of management. Organizations devote considerable resources to the design and implementation of internal control systems, a means of safeguarding assets. A key element of such systems is the separation of decision-making responsibility from record keeping responsibility [Arens & Loebbecke, 1984]. This separation manifests itself in the controllership function. Among the controller's responsibilities is the use of internal audit and accounting control procedures to ensure the validity of information, and to prevent and detect theft and defalcation [Anthony et al. 1989].

This paper examines the controller's role in designing optimal procedures for preventing and detecting employee thefts, when the employee has access to assets at multiple locations. The multiple location problem has been studied in the accounting and auditing literature by Anderson and Young [1988], Hansen [1993], Newman, et al. [1993, 1994], Rhoades [1994], and others. A typical result is that the multiple location problem does not decompose into a series of independent single location problems. Optimal verification strategies for one location tend to depend on characteristics inherent to other locations.

This paper concentrates on the controller's ability to learn through sequential verification. Amershi, Demski, and Fellingham (ADF) [1985] motivate the study of control problems in a sequential setting. They examine conditions under which global decision analysis problems decompose into a series of independent local decision analysis problems. They demonstrate that conditions for complete separation are stringent. They also propose that their result applies to auditing and management control settings, where an important aspect of decisions is that they provide information useful in subsequent problems. In other words, learning is an important consideration in these types of problems. One implication is that a full understanding of optimal theft prevention strategies may require consideration of multiple locations because of the verifier's ability to learn from past decisions. However, the ADF analysis does not consider the impact of a strategic opponent. How their results carry over to a strategic setting is unclear because of the opponent's ability to react to the potential for learning by the controller. This matter is of particular concern to controllers and auditors, who monitor the actions of employees. In practice, controllers and auditors have long recognized the potential for employees to anticipate and react to any form of verification. Additionally, this matter is relevant to strategic auditing research. Strategic auditing papers have repeatedly documented the fact that introducing a strategic opponent into the audit model changes its nature, and the inferences drawn (Fellingham & Newman [1985], Fellingham, Newman, & Patterson [1989] and others). See chapter II for an extensive review of such papers.

Multiple locations are relevant in this paper because of uncertainty about the “control environment.” Verification produces information about the effectiveness of controls. Sequential verification allows the controller to modify his strategy location by location. Uncertainty about the control environment is a reasonable assumption. In practice, controllers are responsible for maintaining and designing internal control systems. Typically however, controllers cannot directly observe the effectiveness of their organization’s internal control system. Instead, inferences about the system’s performance are made indirectly, over time, through the use of internal audit procedures.

In this paper, the controller is unsure of the effectiveness of the firm’s internal controls. If the control environment is operating at a high level, it is easier to detect thefts at all locations. If the control environment is operating at a low level, thefts are more difficult to detect. The controller learns about the level of controls each time he obtains the results of an investigation. For instance, if the controller investigates one location and detects theft, he updates his beliefs about whether the control environment is high or low. The controller may use the results obtained from one location to alter his strategy at subsequent locations. However, in this paper there is an opponent; an employee who also acts strategically. The employee can take actions intended to obscure the information generated by sequential verification.

The principal result of this paper is as follows. When there exists uncertainty about the control environment, sequential verification generates information which is potentially useful to the controller. However, there exists an equilibrium in which the

employee destroys the controller's incentives to use the information. In this equilibrium, the employee understands the informational advantage that the controller can obtain through sequential verification. In response, the employee strategically chooses to steal assets in such a way that prevents the controller from using the information in any useful fashion. In other words, I show there exists an equilibrium in which the employee's actions ensure that the controller does not expect to be better off by adopting a sequential verification strategy. This equilibrium is consistent with anecdotal evidence suggesting that employees deliberately engage in activities intended to obstruct or hinder audit procedures.

This result has some practical implications. In particular, it provides evidence suggesting that strategic considerations are of diminished importance in the staffing and timing of investigations. In other words, it may be appropriate for the controller to base decisions about timing and staffing primarily on cost considerations. Anecdotal evidence indicates that cost considerations are in fact the primary basis for these decisions

This result also suggests that commitment by the controller may be useful for creating a demand for the information.<sup>1</sup> If the controller can pre-commit to strategies which violate sequential rationality, he can potentially make the firm better off. One

---

<sup>1</sup> Another way to address this issue is to allow commitment. Commitment permits solutions that are not sequentially rational. Commitment solutions beg the question of how the commitments will be enforced. Enforcement may occur endogenously through reputation building in a multi-period setting, or through exogenous mechanisms such as courts. In the latter case, there are likely costs associated with enforcement. If commitment can be enforced costlessly, then the controller will always be better off by committing to a policy that violates sequential rationality.



extreme form of commitment is full auditing. In the model in this paper, full auditing will completely eliminate employee theft. In this case the firm will incur only the costs associated with verification.

If the controller is facing multiple employees, each with access to a single location, the equilibrium that produced the result described above still exists. However, there exists another equilibrium where it is possible for the controller to be better-off with a sequential strategy if penalties for detected thefts are sufficiently severe. The equilibrium that produces these results assumes the employees will act independently.

A secondary result is that the optimal non-sequential verification plan depends on the controller's beliefs about the control level. The probability of verifying each location is inversely proportional to the expected level of controls.

The remainder of this dissertation is structured as follows. Chapter II provides a literature review. Chapter III introduces the benchmark, non-sequential model. Chapter IV and V allow sequential verification. Chapter IV considers the case where one employee has access to two locations. Chapter V considers the case where there is one employee per location. Chapter VI concludes.

## **CHAPTER II**

### **Literature Review**

#### **2.1 Introduction**

In this chapter, I examine the development of the “strategic auditing” literature. Specifically, I look at work that utilizes game theory to formulate and analyze auditors’ decisions made in the presence of auditees who react strategically to these decisions. I focus on work in which the auditor’s commitment to verification policies is endogenous. This restriction leads me to exclude models utilizing a principal - agent contracting framework (i.e. Demski and Swieringa [1974], Evans [1980], Antle [1982], Baiman, Evans and Noel [1987] ). These models are also game-theoretic in nature. However, the principal difference between these models, and the ones I review is as follows. Principal - agent models take the players’ preferences as given, payoffs are endogenous, and the principal (auditor) is allowed to commit to a given strategy. Optimal contracts in these models require the auditor to commit to some policy or procedure that is not rational ex-post. The enforcement of such commitment is exogenous to the models. In the literature that I label “strategic auditing,” payoffs are exogenously specified, but any commitment by the players must be self-enforcing.

Fellingham and Newman (FN) [1985], is generally regarded as the first main strategic auditing work. Prior to 1985, it was common to model auditor decision problems in a traditional single person decision analysis setting (i.e. Kinney [1975a,

1975b, 1979]. FN illustrated the limitations in this type of analysis by allowing the auditee to strategically react to a given audit plan. Specifically, FN showed that the use of single-person decision theory to estimate audit risk may lead to erroneous assessments because it fails to consider how the auditee will respond to the audit. Additionally, FN emphasized that it is desirable for the auditor to be unpredictable in these types of settings. Unpredictability (operationalized as mixed strategies) is critical in preventing the auditee from undermining audit strategies. Many of the works that followed FN established the usefulness of game theory and randomization in other types of audit settings.

Table 1 presents a taxonomy of relevant strategic auditing works, characterized by topic. In this endeavor, I will likely omit some works. However, the ones included are the most widely recognized and cited.

**Table 1**  
**Taxonomy of Strategic Auditing Literature**

Tax Compliance	Internal Auditing	External Auditing
Graetz, Reinganum & Wilde [1986] Reinganum & Wilde [1986] Beck, Jung [1989] Sansing [1993] Melumad, Wolfson & Ziv [1994] Rhoades [1995]	Anderson & Young [1988] Hansen [1993, 1995] Newman, Park, Smith [1994] Newman, Rhoades & Smith [1995]	Fellingham & Newman [1985] Patterson [1988, 1993] Newman & Noel [1989] Fellingham, Newman & Patterson [1989] Shibano [1990] Matsumura & Tucker [1992, 1995] Hansen & Watts [1993] Noel & Patterson [1994] Bloomfield [1995]

Of course there are other dimensions on which these works might be characterized. For instance, Newman, Rhoades, and Smith [1995] categorize by whether the amount of information available to the auditor and the amount that the employee can misappropriate is variable or fixed.

Following is a summary of the main characteristics, and important results of selected works in each category.

## **2.2 Tax Compliance**

Before reviewing the work in this area, it is important to understand its objectives. In general, this stream of literature is not attempting to develop a comprehensive model of tax compliance. Rather, the authors are trying to develop a conceptual framework to help provide intuition about taxpayer behavior. The first step in such a process is to understand the nature of “game” that is being played. Only when this aspect of tax compliance is understood, do we have a chance to make prescriptive statements about changes in policies. This literature stream is currently in the first stage. Authors are trying to understand which aspects and results of their work are robust to different assumptions about the players and the environment.

Prior to Graetz, Reinganum and Wilde (GRW) [1986], it was common for work investigating taxpayer actions to treat taxing authority policies as exogenous. GRW suggested that the results of these models were not descriptively valid, and documented a need for an interactive theory of taxpayer compliance. They introduced the IRS as a strategic player, allowing it to condition its strategy on the taxpayers’ reports of income.

Their primary result is similar in spirit to FN. In particular, introducing the IRS as a strategic player fundamentally alters the model. When considering changes in penalties, audit costs, or tax rates, the strategic model yields different conclusions than the non-strategic (taxpayer only) model. They also find that both the taxpayer and IRS often choose interior probabilities for mis-reporting and verifying respectively. Again the importance of unpredictability is emphasized. One limitation of the model is that the taxpayer's income can take on only one of two values (high or low) and that the IRS is restricted to a costly binary choice (audit or not audit).

Reinganum and Wilde [1986] extends GRW by allowing a continuum of taxpayer incomes. Also the IRS chooses a probability of verification, with the audit cost increasing in the probability. They find that the conclusions in GRW are fundamentally unaltered.

Beck and Jung (BJ) [1989], modelled the tax compliance problem in a novel way. Previous work supposed that taxpayers had private information about their income levels, and could choose to mis-report this information. BJ suggested that in actuality, many taxpayers are uncertain as to the true tax liability associated with their private information. They must report their tax liability, which involves making a decision about how aggressive they should be in issuing the report. In this spirit, the taxpayer is not really engaging in fraudulent behavior. Nevertheless, aggressive reporting may trigger an audit. The audit, in turn may reveal that they were too aggressive (ex-post); in which case they are subject to penalty. Of course, the probability of an audit is determined

endogenously. An important result from this study deals with the penalty structure imposed for misstated tax liability. Specifically, under fixed penalties, more uncertainty may induce more aggressive reporting. Conversely, under proportional penalties, more uncertainty induces less aggressive reporting. BJ suggest that this result provides insight regarding the effects of changes in tax law on taxpayer behavior.

Continuing this line of inquiry, Melumad, Wolfson, and Ziv (MWZ) [1994] investigated behavior when taxpayers who were uncertain about their liability could hire a third party preparer. MWZ suggest that hiring a third party preparer can help reduce taxpayer uncertainty, but it also has a signaling effect. That is, the decision to hire a third party preparer may provide information about a taxpayer's situation. If the signal is informative, the taxing authority may use it as part of its audit strategy. MWZ find that hiring a third party preparer is informative.

They extend their analysis by investigating whether the taxing authority should provide incentives for third party preparation by issuing tax credits for preparation fees. They find that the taxing authority will want to allow the credit only if it can price discriminate. That is, credits will only be granted under specific circumstances (i.e. nobody who is audited will receive a credit). Furthermore, if the taxing authority is required to provide either a uniform credit available to everyone, or no credit at all, the authority will prefer no credit at all.

### **2.3 Internal Auditing**

In this next section, I consider the development of the internal auditing literature. In this literature, the internal auditor is primarily concerned with preventing and detecting fraud or theft. Furthermore, his incentives are assumed to be aligned with the firm. He does not issue an opinion on the financial statements, nor is he penalized for failure to uncover errors. Another characteristic of this literature, is that it tends to deal with multiple locations and resource allocation decisions. The notion behind multiple locations is that firms typically have numerous assets. These assets may have different characteristics. Thus, the auditor and auditee may have incentives to treat these assets (locations) differently. This stream recognizes that the study of multiple locations is important to a full understanding of the nature and objectives of audit tests.

This literature recognizes that the distribution of audit resources is an important dimension of the audit problem. It attempts to show how the introduction of multiple auditable units may distort decisions, and it attempts to produce results that resemble observed sampling procedures.

Anderson and Young (AY) [1988] were the first to model the internal auditor's problem in a multiple location setting. In the spirit of FN and GRW, AY stressed the importance of allowing the auditor and auditee to endogenously form beliefs about each other's actions. Additionally, they emphasized the importance of "unpredictable" behavior by the auditor. In their setting, however, unpredictability was represented by more than just mixed strategies. In particular, they found it optimal for auditors to treat

locations with identical characteristics differently. Of course, the study of multiple locations made this insight possible. Strategically distorting the probability of auditing identical locations helped prevent thefts. In their model AY can actually eliminate theft completely. This result hinges on the fact that the auditor has a fixed budget which must be spent on auditing. Essentially they allow the auditor to commit to a specified (positive) level of auditing. Of course, if the auditee does not steal, it is not a best response to carry out the investigation.

Newman, Park, and Smith (NPS) [1994], extended AY. They allowed the players to choose from a continuum of strategies, and they also determined the optimal budget levels. (AY took the budget as exogenous, and had discrete choices for theft and verification). Furthermore, NPS adopted a discovery sampling perspective. That is they assumed that the internal auditor is interested in detecting a single instance of theft. Upon discovery, all other thefts are costlessly revealed. This may occur due to the unraveling of a systematic scheme for defrauding the firm, or it may occur if the authorities are called in, and subsequently bear the cost of a criminal investigation. NPS find that increasing penalties does not necessarily reduce the amount of testing required, and that the optimal budget size is not monotonic in firm size (number of locations).

-- Hansen [1993] presents the multiple location problem as one in which an internal auditor chooses the probability of investigating accounts receivable line items. In this stylized setting, he shows that when employees strategically create billing errors in accounts receivable, there exist auditor strategies that resemble the commonly observed



practices of physical units sampling and dollar width sampling. This result, while highly model-specific, is an important one. It suggests that despite restrictive assumptions about payoffs and rationality, game theory has the potential to produce results that are descriptive<sup>1</sup> of actual audit practices. Conversely, it suggests that when observed practices such as stratified sampling are subjected to economic scrutiny, they emerge as optimal solutions. At a minimum, this should encourage academicians to continue to study auditing in a game theoretic setting.

## **2.4 External Auditing**

External auditing has received the most attention in the strategic auditing literature. In this line of inquiry, the preferences and objectives of the auditor tend to differ from those characteristic of internal auditing research. Specifically, in this stream the auditor is typically required to issue an opinion on a financial report. Often, he is required to decide whether or not to obtain more (costly) evidence prior to issuing a report. The auditor in these settings is concerned with penalties for incorrect decisions. These penalties may be lawsuits for incorrectly issuing “clean” opinions, or reputation losses for incorrectly issuing “qualified” opinions.

Beyond the differences in objectives, external auditing research distinguishes itself by the questions it addresses. Two themes are common in many of these papers. First, many papers investigate the auditor’s incentives for acquiring information. Second,

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<sup>1</sup> Game theory also has the potential to produce normative results. However, the state of the literature is such that researchers are still trying to identify which descriptive results are robust, and which results are model specific.

many papers investigate whether standard methods for assessing audit risk hold up in game theoretic models.

FN can be characterized as external auditing. Additionally there are numerous other works, as evidenced by the table above. Following is a strategic sample of these works.

Fellingham, Newman, and Patterson [1989] focused on the auditor's incentives for information acquisition in a strategic setting. In their model, an auditee chooses an error rate to seed in an account balance. The auditor chooses whether to accept the balance, reject it, or seek additional evidence based on a signal generated by an accounting system. Again, randomized strategies are an equilibrium outcome. When an employee can influence the rate of errors inherent in an account balance, the auditor's best response is to randomize between extending procedures to find out the true error rate and accepting the balance. FNP find this appealing because it endogenously introduces noise into the audit environment. Their results are similar in nature to stratified audit sampling. Thus, they claim to initiate a descriptive theory of audit strategy, emphasizing the interactive nature of the auditor - auditee relationship.

Newman and Noel (NN) [1989] developed a similar model. Their goal, however, was to investigate how changing the auditor's and auditee's payoffs will impact the existence and magnitude of error rates, undetected errors, and incorrect rejections of account balances. In their model, the auditee chooses whether to seed a material error in the account balance in question. The auditor, has a set of priors on the probability that an

error exists (his priors are endogenous). He then observes a signal causing him to update his beliefs about the existence of an error. Based on his updated beliefs, he either accepts or rejects the account balance.

NN find that changing the auditor's payoff has unambiguous impacts on error rates, undetected errors, etc. Conversely, changing auditee payoffs leads to ambiguous results. NN suggest that from a policy standpoint, it may be more fruitful to attempt to modify auditor behavior as a means of implementing policy changes. NN also suggest caution when investigating (empirically) the effects of policy or technology changes on the level of "errors" in a population. Their comment is based on the notion that policy changes impact both detected errors (observable) and actual errors (unobservable). Thus, for example, requiring more extensive audit testing has two effects. First it will likely decrease the amount of errors. Second, it will increase the probability of detecting errors, given they have occurred. The impact on detected errors, then, may either increase or decrease, depending on the interaction between the two main effects.

Patterson [1993] continued to push the idea of strategic audit sampling. She showed that in a strategic setting, the components of audit risk (inherent risk, control risk, and detection risk) are not conditionally independent. Conditional independence is assumed in many auditing texts and in many risk-based audit approaches utilized in practice. Her paper considers a hidden action setting in which the auditee intentionally seeds errors into a population of transactions (possibly through theft). The auditor then observes evidence, and issues a report based on the evidence. Patterson allows the

auditor to determine how much evidence to observe, by choosing a sample size. She shows that audit risk is not strictly decreasing in sample size, which counters the notion that materiality and sample size are inversely related.

Her paper is important because it suggests that auditors may not be properly assessing audit risk when planning engagements. Given the volume of litigation associated with audits, this result suggests that auditors may need to rethink their strategies.

One limitation of the external auditing works cited is that they do not consider the possibility of strategic financial reporting. There either is no financial reporting, or the report is modelled as a random variable, whose distribution is systematically affected by auditee decisions. Recognizing this limitation, Noel and Patterson [1994] and Hansen and Watts [1993] look at issues relating to strategic financial reporting.

Hansen and Watts (HW) [1993] acknowledged the desirability of investigating the auditor's incentives to acquire information. They noted, however, that often, the information sought after is produced by management. Noting empirical evidence of "report management", HW ask whether it is appropriate for auditors to rely on unaudited management reports when planning an audit.

In their model, an auditee privately observes a noisy signal about firm performance. He then issues a report based on this signal. His payoff is exogenously tied to the report. The auditor may accept the report, or conduct a costly audit which reveals true firm performance. HW examine two settings; one in which the auditee does not

strategically report, and one in which he does. The main result is that the report is useful to the auditor in planning tests, despite the possibility of report management. Additionally, when comparing strategic and non-strategic scenarios, HW find that any report audited in a non-strategic setting will be audited in a strategic setting. Also, in a strategic setting additional reports may be audited. However, HW note that the mapping from private information to reports changes with the setting, so the auditor, while testing the same reports, may not be testing the same underlying signals.

A common thread in all of the papers cited is that the models assume that each player makes accurate inferences about the other player's strategy. They are silent on how these inferences are made. FN recognized that it may be difficult for the players to arrive at these inferences. In a recent paper, Bloomfield [1995] investigates this decision process. Specifically, he does not assume that the players will necessarily form accurate expectations about each other's behavior. Instead, he assumes that the players will construct their beliefs through the iterated elimination of dominant strategies, a process called *rationalization*. He shows that in some cases a Nash equilibrium is achieved after only a few iterations. In others, however, even an infinite number of iterations will not produce a Nash equilibrium. In these settings, Bloomfield claims the use of Nash equilibrium as a solution concept will generate misleading conclusions about auditor and auditee behavior. He finds that rationalization tends to produce Nash equilibria only when the manager's best response function is relatively insensitive to changes in the auditor's expectations.

## 2.5 Conclusion

It seems reasonable to ask: After a decade of strategic auditing research, what have we learned? Based on the evidence cited above, several conclusions can be drawn.

First, despite restrictions on solutions, and simple models, game theory has been successful at producing results which are intuitive and descriptive of observed practices. It has justifiably replaced single-person decision theory as one fundamental method of academic inquiry into auditing questions.

One example of an intuitive result is that strategies are often consistent with stratified audit sampling. Pure random sampling, which is appropriate in games against nature is inappropriate in strategic settings. A second example of an intuitive result is the desirability of being unpredictable. In practice, auditors tend to be secretive about announcing the locations of audits, revealing materiality levels etc. Many of the citations require mixed strategy solutions, or interior solutions. This is equivalent to the auditor not announcing test procedures.

As mentioned above, strategic auditing research has helped academics establish economic results that explain observed practices. The true value of this research, however, may be its ability to produce counter-intuitive results. The lesson to be learned from this work is to beware when making policy changes. It is difficult to casually predict results because of interactions between players. Following are two brief examples of how adding a strategic opponent has produced unexpected results.

In many audit texts, and many audit planning manuals, audit risk is defined as the product of inherent risk, control risk, and detection risk. Several papers have shown that with strategic auditees, the audit risk should not be expressed this way because the three components are not conditionally independent.

Also, strategic interactions tend to produce unusual results relating to changes in penalties. Specifically, raising penalties on auditors for audit failures, and changing penalty structures for tax evasion, may have effects opposite of those desired.

So, it seems that strategic auditing research is an appropriate forum for investigating audit problems. One remaining question is, what should be done in the future?

Thus far, most strategic auditing models have been single period models. These models have had some success in producing interesting and intuitive results. However, we know that these audit games do not occur in a vacuum. The audit environment is inherently multi-period in nature. Both auditors and auditees build reputations. Reputations may impact how these parties deal with each other. Future research needs to investigate how single period results hold up in a multiple period setting. It is likely that the introduction of multiple periods will produce additional insights. Multiple periods may also produce results inconsistent with the single period models. In such cases, researchers will need to determine which, if any, of the results has the most external validity.

One of the problems with game theory, is that it is useful for simple games, but as the games become more complex (additional periods, more choices), strategies become difficult to represent, and additional refinements are needed to determine reasonable solutions. It is these restrictions have hindered researchers' ability to examine things such as reputation building in multi-period settings. It is likely that progress on this front will be slow. New insights may likely come from approaches that make multiple period analysis more tractable. Concepts such as bounded rationality may be useful.

It should be noted that this literature does not consider the ability of contracting to act as a substitute or complement for auditing. There exists a solid base of research using contracting (and monitoring) to control the behavior of employees and managers. At a minimum these two areas need to be reconciled. That is, researchers need to determine how contracting, and strategic auditing should be jointly employed to efficiently solve problems. Given the tractability problems associates with multi-period strategic auditing models, it may be fruitful to consider the possibility of contracting in order to make the models more tractable.

More research needs to be undertaken examining the similarities and differences between theft and fraud. Many papers claim to be sufficiently general to allow readers to interpret auditee actions as either theft or fraud. This is fine because these two actions do have many similar characteristics. However, intuition suggests that these actions are fundamentally different. Theft relates to items that have intrinsic value (pilferage of inventory, kiting checks, shirking). Gains to fraudulent behavior tend to be more indirect



(compensation contracts tied to performance, stock options, etc.). This suggests that procedures designed to prevent and detect theft may not be appropriate for preventing and detecting fraud, and vice-versa. Understanding these differences is essential to a full understanding of the audit problem.

Finally, attention should continue to be directed examining how players arrive at their decisions. Research in this area may help researchers determine when it is appropriate to use Nash equilibrium and its refinements as a solution concept. Of course, in situations in which Nash may not be appropriate, researchers should examine the applicability of alternative solution concepts.

The preceding review has been shaped by my biases, and is by no means complete. Despite these limitations, I hope it presents fairly the current state of strategic auditing research.

## CHAPTER III

### Non-Sequential Verification

#### 3.0 Model Setup

Assume a firm is endowed with assets at  $n$  locations. The locations differ across three dimensions; asset value, asset specific investigation costs, and asset specific detection probabilities. Additionally, the firm's internal control system further influences detection probabilities. Detection probabilities are greater across all locations when controls are effective than they are when controls are ineffective. The effectiveness of the internal control system is operationalized as follows: There is a control parameter  $\alpha \in (\alpha_H, \alpha_L)$  common to all locations. The control parameter influences the probability of detecting thefts by interacting with the detection probability parameter. For instance, high controls ( $\alpha_H$ ) increase detection probabilities across all locations relative to low controls ( $\alpha_L$ ).

An employee maintains physical custody of the firm's assets. This job requires no effort on the part of the employee. He does, however, have the ability to steal assets from any of the  $n$  locations.

I assume that the firm employs a controller, whose responsibility is to maintain proper stewardship of the assets. In doing so, I assume away any agency problems between the firm and its controller. I do this so that I can focus on the design of optimal procedures to prevent and detect employee thefts as opposed to contracting arrangements designed to motivate the controller, or to prevent collusion between the agents<sup>1</sup>.

The controller must establish and carry out a strategy for verifying the asset values at each location. The controller potentially considers the asset values, detection risks, verification costs, and the control environment when establishing this strategy. Again, the controller does not observe  $\alpha$ .

### 3.1 Assumptions

In this model (model S1),  $\alpha$  is unobservable to both players. Several interpretations of  $\alpha$  are possible in this context. For instance,  $\alpha$  might represent the effectiveness of internal controls due to environmental factors. Complex transactions may obscure both parties' abilities to observe the effectiveness of internal controls.

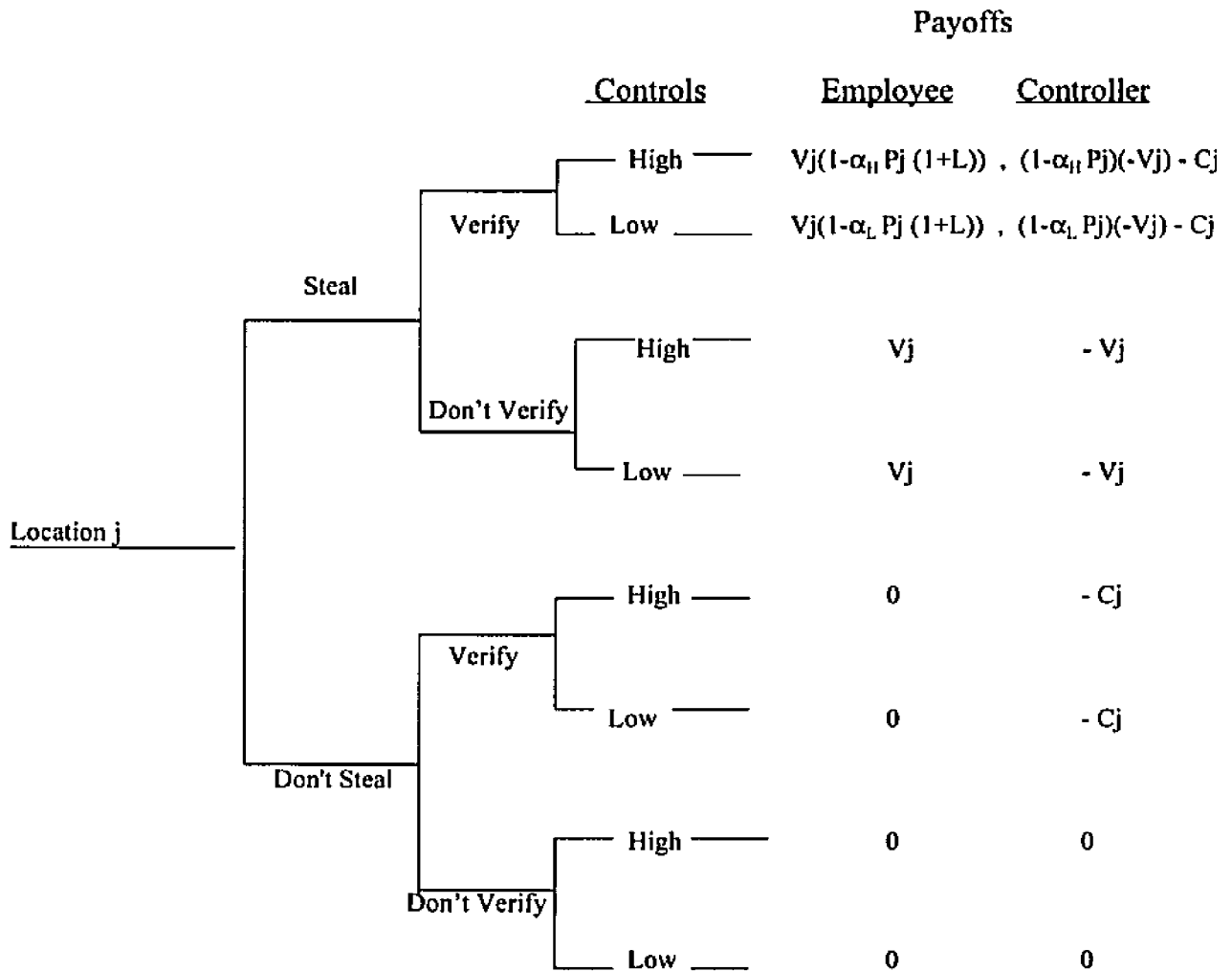
Another assumption of model S1 is that the controller must employ a non-sequential verification plan. I assume non-sequential verification to establish a benchmark for comparison with a sequential plan. This type of plan has two

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<sup>1</sup> These assumptions are readily acknowledged as limitations of this work. They also present opportunities for future research. In the spirit of ADF, incentive issues may not separate completely from the problem considered in the paper. In particular, contracting arrangements between the owners and employee might be a substitute or complement for verification. Furthermore contracting arrangements that align the incentives of the controller and the firm may spill over into the verification problem.

interpretations. First, one might think of a non-sequential plan as one where the controller selects which locations to verify, and then physically performs the verifications simultaneously. Another interpretation is that a non-sequential plan is a particular form of commitment. Specifically, the controller commits to ignore any information generated (about internal controls) by sequential verification in this model. However, I do not examine how the controller can credibly commit in this setting. As will be illustrated later, non-sequential verification may sometimes outperform sequential verification.

In model S1, the employee forms and carries out his theft strategy. Next, the controller chooses and implements a verification strategy. Refer to figure 1 for an extensive form description.



**Legend:**

- $q$  = Probability that control environment is high
- $V_j$  = Asset value at location  $j$
- $C_j$  = Verification cost at location  $j$
- $\alpha_{Lj} P_j$  = Probability theft is detected at location  $j$ , given theft and verification occur, and control effectiveness is low
- $\alpha_{Hj} P_j$  = Probability theft is detected at location  $j$ , given theft and verification occur, and control effectiveness is high
- $L$  = Penalty rate for detected thefts

**Figure 1**  
**Extensive Form Game Tree**

### 3.2 Controller Strategies and Payoffs

The controller must design a plan for verifying the status of the assets at the  $n$  locations. His objective is to minimize the firm's expected costs, which include verification costs and losses from theft. If a theft is detected, the firm reclaims the asset from the employee. I assume the following detection technology:

$\alpha_i P_j$  = probability of detecting theft at location  $j$  given;  
 controller investigates location  $j$ ,  
 theft has occurred at location  $j$ ,  
 and control environment is  $\alpha_i$ .

$C_j$  = fixed cost of investigating location  $j$

where,  $0 < P_j < 1$  and  $C_j > 0 \forall j = \{1, 2, 3, \dots, n\}$ , and  $\alpha_i \in \{\alpha_{1i}, \alpha_{1i}\}$ .

The controller designs a plan specifying the probability of verifying the asset values at each location.

$[A_j]$  = (probability of verifying asset value at location  $j$ )

$[1-A_j]$  = (probability of not verifying asset value at location  $j$ )

Following is an illustration of how to calculate the firm's payoffs (Figure 1). Suppose  $\alpha_i = \alpha_{1i}$ , the employee has stolen assets from Location 1, and the controller has chosen to investigate Location 1. In this case there is a  $(\alpha_{1i} P_1)$  chance that the theft will be detected, in which case no losses will occur. There is a  $1 - \alpha_{1i} P_1$  chance that the theft will go undetected, resulting in a loss of  $V_1$ . The firm incurs a cost of  $C_1$  to investigate Location 1. Combining these, the following expected payoff is obtained:  $(1 - \alpha_{1i} P_1)(-V_1) - C_1$ .

One assumption is important to note. The firm only recovers thefts it has detected. This assumption differs from assumptions made in Newman et al. (1993, 1994) in which the detection of one theft costlessly reveals all other thefts. This assumption potentially creates a demand for sequential verification.

### 3.3 Employee Strategies and Payoffs

The employee's objective is to maximize his expected payoff. His wealth is increased by the value of the stolen assets ( $V_j$ ). If caught stealing, the employee must return the asset(s) and incur a penalty ( $L$ ) proportional to the value of all detected thefts. For instance, if the employee is caught stealing from Location 1, he is penalized  $V_1 * L$ . This penalty is assumed to be a dead-weight loss, which does not accrue to the firm<sup>2</sup>.

The employee chooses probability stealing the assets at each location:

$[T_j]$  = (probability of stealing from location  $j$ )

$[1 - T_j]$  = (probability of not stealing from location  $j$ )

Following is an illustration of how to calculate the employee's payoffs (Figure 1).

Suppose the employee has stolen the assets at Location 1, the controller has chosen to investigate Location 1, and  $\alpha_i = \alpha_H$ . In this case there is a  $\alpha_H P_1$  probability that the theft will be detected, resulting in a loss of  $V_1 * L$ . There is a  $1 - \alpha_H P_1$  probability that the theft

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<sup>2</sup> The penalty is assumed to be a dead weight loss to ensure that the controller's motive is strictly one of theft prevention. If the penalty did accrue to the firm, then the controller might have incentives to generate revenue through excessive verification. This is analogous to the Highway Patrol setting up speed traps at the end of the month **not** to deter speeding and prevent accidents, but to generate revenue to meet State budgets.

will go undetected, resulting in a gain of  $V_1$ . Combining these, the following expected payoff is obtained:  $V_1 - (\alpha_{II} P_1 V_1)(1+L)$ .

Before presenting the solution to model S1, I will place some reasonable restrictions on the parameters. First, since I am interested in cases in which the potential losses to the employee are sufficient to deter theft when the controller investigates, I assume  $V_j * \alpha_L P_j * L > (1 - \alpha_L P_j) * V_j$  for all  $j$  locations. This implies that if the employee chooses to steal and the controller chooses to investigate, the employee's expected losses exceed his expected gains, even if the control environment is low. Second, I assume that even if  $\alpha_L < \alpha_{II}$  obtains, the expected benefits of detecting a theft exceed the costs of investigating ( $\alpha_L P_j V_j - C_j > 0$ ).

### 3.4 Solution

Since both players choose their strategies without knowing the other's choice, model S1 is equivalent to a simultaneous move game. Thus, Nash equilibrium is the concept employed to determine the equilibrium solution.

#### Proposition 1

The Nash equilibrium solution to this game is:

a) Controller Strategy:  $A_j = \frac{1}{\bar{\alpha} P_j (1+L)} \quad \forall j = \{1, 2, \dots, n\}$

b) Employee Strategy:  $T_j = \frac{C_j}{\bar{\alpha} P_j V_j} \quad \forall j = \{1, 2, \dots, n\}$

where  $\bar{\alpha} = q\alpha_{II} + (1-q)\alpha_L$

Proofs of all propositions and corollaries are in the appendix.



Three observations about this solution are noteworthy. First, the controller's plan calls for all locations to be investigated with positive probability. Similarly, there is a positive probability that the employee will steal from all locations. This is true even if there is a very high probability that thefts will be detected with an audit. The relative or absolute detection risk associated with the locations does not influence this result.

Second, the controller's plan calls for high-risk (lower  $P_j$ ) accounts to be investigated with higher probabilities than low-risk accounts. With a proportional penalty structure,  $P_j$  is the only location specific parameter in the controller's strategy. Conversely, the employee may or may not be more likely to steal from high-risk accounts. This is because all three location specific parameters are involved in determining the employee's strategy. As is easily seen in proposition 1, the ratio  $\frac{C_j}{P_j V_j}$  determines the ordering of theft frequencies across locations. Thus the location with the highest cost/benefit ratio (for the controller) will be targeted most frequently by the employee.

Third, both players treat all locations in the same fashion. That is, the same generic strategy is applied to all locations. The final probabilities of theft and verification are determined by substituting parameter values into the strategies.

### 3.5 Comparative Statics

In this section, I introduce one new term. Expected organizational loss (EOL) is the amount the firm expects to lose due to thefts and investigation costs. The firm's EOL based on the equilibrium in proposition 1 is  $\frac{C_j}{\alpha P_j}$ .

**Table 2**  
**Comparative Statics - Non-Sequential Verification**

		$\partial P_j$	$\partial V_j$	$\partial C_j$	$\partial \alpha_H$	$\partial \alpha_L$	$\partial q$	$\partial L$
Model SI	$\partial A_j$	-	0	0	-	-	-	-
	$\partial T_j$	-	-	+	-	-	-	0
	$\partial EOL$	-	0	+	-	-	-	0

Note that  $\frac{\partial EOL}{\partial L} = 0$ . This suggests that there is no marginal benefit for increasing the penalty for theft. The entity responsible for setting the penalty rate,  $L$ , merely needs to ensure that the penalty for stealing exceeds the benefits when the controller investigates. Additional penalties have no impact. The intuition behind this result lies in the solution concept. To maintain a Nash equilibrium in mixed strategies, the employee must be indifferent between stealing and not stealing. If the penalty for theft increases, then the amount of verification must decrease such that the effects of changing the penalty are mitigated. The result is that the employee's behavior is unaffected.

## Chapter IV

### Sequential Verification (Single Employee)

#### 4.1 Assumptions

In this section I extend my analysis of the model. Specifically, I allow the controller to adopt a sequential verification strategy. Recall that the control parameter ( $\alpha$ ) is constant across locations, but the controller does not know whether  $\alpha = \alpha_H$  or  $\alpha = \alpha_L$ . In a game against nature (i.e. the controller is searching for random errors), sequential verification will always be better than non-sequential verification. The controller, by verifying non-sequentially, leaves useful information on the table. However, in this setting, the employee is acting strategically. So it is possible that the employee will devise a strategy that undermines the controller's ability to use the information learned about  $\alpha$  via sequential verification. I will show that this is precisely what happens.

Because of tractability concerns, I make some simplifying assumptions. First, there are two locations. Both locations have identical asset values, detection probabilities, and verification costs. Because the locations are identical, the controller should be indifferent as to which location is selected first. Ex-ante, each location has a 50% chance of being the first location selected by the controller. Refer to the first

location selected by the controller as “Location 1” and the second as “Location 2”. The employee does not know which location will be Location 1, or which will be Location 2.

In this section, the controller’s strategy space changes. Specifically, The controller’s Location 2 strategies can be conditional on the outcome of the Location 1 investigation. For instance,  $[A_2 | A_1D_1]$  is the probability of investigating Location 2, given Location 1 was investigated, and a theft was detected.  $[A_2 | A_1ND_1]$  is the probability of investigating Location 2, given Location 1 was investigated, and no theft was detected. Finally,  $[A_2 | NA_1]$  is the probability of investigating Location 2, given Location 1 was not investigated.

I assume the employee arbitrarily chooses one location (Location “a”) and steals with probability  $T_a$  from that location.  $T_b$  is similarly defined for the other location. The controller does not know which is Location “a”, or which is Location “b”, and cannot differentiate between them ex-ante (recall they are identical). Again,  $T_a$  may or may not be equal to  $T_b$ . Since the employee may choose different probabilities of theft for each location, the controller is concerned about two things; whether he is at location “a” or location “b” and whether controls are effective ( $\alpha_{1f}$ ) or ineffective ( $\alpha_{1l}$ ). (If the employee chooses  $T_a = T_b$ , the only thing the controller is concerned with is the effectiveness of controls.)

The solution concept in this game is Bayes - Nash sequential equilibrium. The employee chooses the probability of stealing from Locations “a” and “b”, and the controller chooses the probability of verifying at Location 1, and the probability of

verifying at Location 2, given the outcome from the first location. Again, the employee does not know which location will be selected first. Conversely, the controller does not know which location was selected as Location “a” by the employee. His strategy for the second location must be sequentially rational and consistent with his beliefs about which location he is at, and the effectiveness of the internal controls.

## 4.2 Solution

In this section I show that the employee can choose  $T_a$  and  $T_b$  such that the controller will not benefit from any information obtained about the controls. Lemma 1 and Proposition 2 summarize.

### Lemma 1

Despite the fact that the two locations are identical, there is no equilibrium in which the employee chooses  $T_a = T_b$ .

### Proposition 2

An equilibrium where  $T_a \neq T_b$  is characterized as follows:

$$\text{i) } T_a = \frac{c}{\bar{\alpha} p v} \left\{ 1 + \sqrt{\frac{k - \bar{\alpha}^2}{k}} \right\} \quad \text{and} \quad T_b = \frac{c}{\bar{\alpha} p v} \left\{ 1 - \sqrt{\frac{k - \bar{\alpha}^2}{k}} \right\}$$

where  $k = q \alpha_{11}^2 + (1-q) \alpha_{1L}^2$  and  $\bar{\alpha} = q \alpha_{11} + (1-q) \alpha_{1L}$ .

$$\text{ii) } [A_2 | A_1 D_1] = [A_2 | A_1 N D_1] \equiv [A_2 | A_1] \text{ and}$$

$$A_1 + ([A_2 | A_1] A_1 + [A_2 | N A_1] (1 - A_1)) = \frac{2}{\bar{\alpha} p (1 + L)}, \text{ or}$$

$$\text{iii) } A_1 = 0 \text{ and } [A_2 | N A_1] = \frac{2}{\bar{\alpha} p (1 + L)} .$$

### 4.3 Analysis

There are several noteworthy points about Proposition 2 and Lemma 1. First there are an infinite number of equilibrium solutions. The controller is by-and-large free to allocate verification probabilities as he wishes. As long as he adheres to the restrictions in parts i and ii, all solutions are payoff equivalent. The controller loses  $(\frac{-2c}{\bar{\alpha}p})$  in expectation. This expected payoff is equivalent to the expected payoff in the non-sequential model (S1). This raises an interesting point. In this equilibrium, the controller gains no strategic advantage by moving in sequence. Proposition 2 is consistent with the idea that strategic considerations are of diminished importance in the timing and staffing of investigations. The solution in Proposition 2 allows the controller to mimic the non-sequential strategy, and earn the same expected payoff. For example, suppose the controller has many locations to investigate, and a deadline for completion. Proposition 2 permits the controller to staff this investigation heavily, possibly sending multiple agents to the locations simultaneously. Despite the fact that information learned about controls will not be used, the controller does not expect to be worse off.

Another interpretation of this solution deals with external audit settings, and interim testing. Often, external auditors choose to do compliance testing and interim substantive tests prior to year-end. Anecdotal evidence suggests that decisions regarding the timing of tests is based primarily on cost considerations. Taking some liberties with the abstract nature of this model, this equilibrium is consistent with that evidence. The auditor gains no informational advantage with sequential verification (interim testing).

The employee's actions drive this result. They can be interpreted as follows: The employee exploits the fact that the controller's actions must be sequentially rational. He perturbs the theft probabilities in order to mitigate what the controller learns about the control system's effectiveness. By carefully choosing different theft probabilities at each location, the employee will cause the controller to revise his beliefs about which location he is at (a or b) in a way that precisely offsets his revisions about the level of controls. Thus, the employee can make the controller indifferent between verifying and not verifying at each location. An important aspect of this equilibrium is that the controller is still indifferent between verifying and not verifying at the second location irrespective of whether or not he detected a theft at the first location. The punch-line is that in this equilibrium<sup>1</sup>, when facing a strategic opponent, the information that the controller obtains regarding the effectiveness of controls is essentially useless. This indicates that it might be useful for the controller to find ways to commit to use the information in ways which are not sequentially rational.

Note that this equilibrium is very similar to the non-sequential equilibrium. The controller verifies with total probability  $\frac{2}{\bar{\alpha} p (1 + L)}$ . How he distributes the probability is arbitrary except for the fact that he must choose  $[A_2 | A_1 D_1] = [A_2 | A_1 ND_1]$  or  $A_1 = 0$ . In the non-sequential equilibrium, the controller verifies both locations with probability  $\frac{1}{\bar{\alpha} p (1 + L)}$ . Additionally, in the non-sequential equilibrium, the employee steals with

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<sup>1</sup> Recall, however, that I have not proved that this is the unique equilibrium in the model.

probability  $\frac{c}{\bar{\alpha} p v}$  from both locations. In the equilibrium above, the employee chooses

the same base probability for both locations ( $\frac{c}{\bar{\alpha} p v}$ ), but strategically reduces theft at one

location and increases theft at the other location. The alteration is related to the amount of

uncertainty about the control environment  $\{1 \pm \sqrt{\frac{k - \bar{\alpha}^2}{k}}\}$ . Hereafter, I refer to

$\delta \equiv \sqrt{\frac{k - \bar{\alpha}^2}{k}}$  as the “distortion term”. This term ( $\delta$ ) quantifies the magnitude of the

departure from the base probability of theft. Note that  $\delta$  is actually composed of three

“primitive” parameters;  $\alpha_H$ ,  $\alpha_L$ , and  $q$ . Thus,  $\delta$  can be expressed as

$$\sqrt{\frac{q(1-q)(\alpha_H - \alpha_L)^2}{\alpha_L^2 + q(\alpha_H^2 - \alpha_L^2)}}.$$

Comparative statics on  $\delta$  produce the following:

**Table 3**  
**Comparative Statics - Sequential Verification, Single Employee**

	$\frac{\partial \delta}{\partial \alpha_H}$	$\frac{\partial \delta}{\partial \alpha_L}$	$\frac{\partial \delta}{\partial q}$
Sign	+	-	+ or -

These changes are intuitive. Holding all else constant, increasing  $\alpha_H$  increases the spread between  $\alpha_H$  and  $\alpha_L$ . This suggests increased uncertainty; implying the magnitude of revisions about the level of controls will increase. Thus the employee must increase

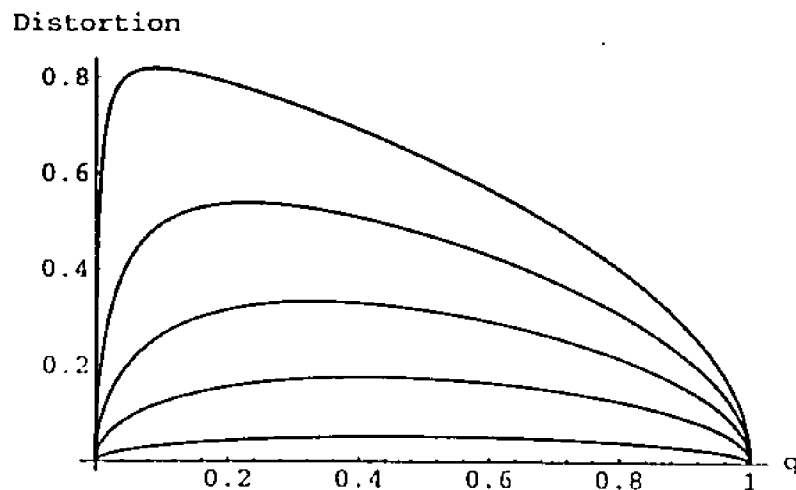


the distortion term to offset the revisions. Conversely, increases in  $\alpha_L$  reduce uncertainty by closing the spread. As such, less distortion is required to offset revisions about controls. Finally, changes in  $q$  have an ambiguous effect on  $\delta$ . This is because there is a unique interior value of  $q$  that maximizes the distortion term. As  $q$  departs from this point in either direction, distortion is decreased. At the limit we have:  $\lim_{q \rightarrow 0^+} \delta = 0$ , and

$\lim_{q \rightarrow 1^-} \delta = 0$ . This suggests that as uncertainty is eliminated (one realization of  $\alpha$

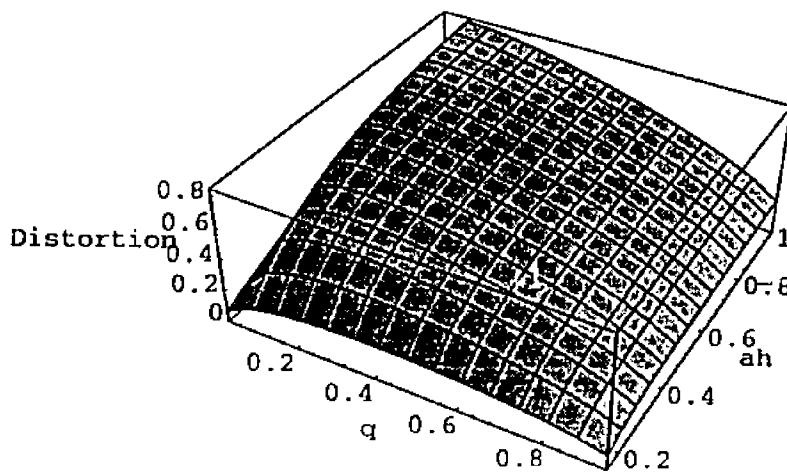
becomes certain) the need to distort the theft probabilities is also eliminated.

It should be noted that the magnitude of  $\delta$  is jointly determined by  $q$ ,  $\alpha_H$  and  $\alpha_L$ . The comparative statics above hold two parameters constant. To understand how the parameters work together, consider the following figures:



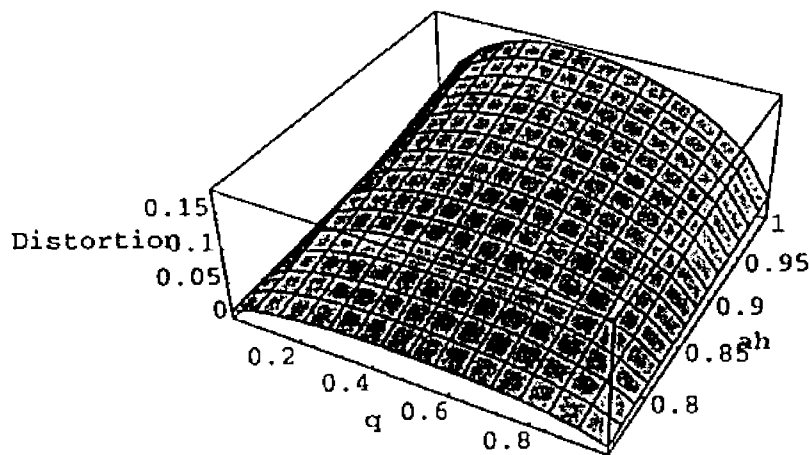
**Figure 2**  
Distortion as a function of  $q$  ( $\alpha_H = 1$ )

Figure 2 holds  $\alpha_{i1}$  constant and depicts  $\delta$  as a function of  $q$ . Each successive plot is for a smaller value of  $\alpha_{i1}$ . As the difference between  $\alpha_{i1}$  and  $\alpha_{i2}$  increases, the magnitude of the distortion increases, and the plots become skewed toward  $q = 0$ . This suggests that when there is little uncertainty about  $\alpha$ , changes in  $q$  have a minimal impact on distortion. As uncertainty about  $\alpha$  increases, distortion becomes much more sensitive to changes in  $q$ , and the region over which  $\delta$  is decreasing in  $q$  gets larger.



**Figure 3**  
**Surface Plot of Distortion as a function of  $\alpha_{i1}$  and  $q$  ( $\alpha_{i2} = 0.1$ )**

Figure 3 is a surface plot of distortion as a function of both  $q$  and  $\alpha_{II}$ . In this plot, distortion is strictly increasing in  $\alpha_{II}$ . The plot, however is skewed. Large  $q$ 's and small  $\alpha_{II}$ 's produce minimal distortion. As  $q$  decreases and  $\alpha_{II}$  increases distortion increases. Once  $q$  becomes sufficiently small, however, distortion decreases very rapidly.




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**Figure 4**  
**Surface Plot of Distortion as a function of  $\alpha_{II}$  and  $q$  ( $\alpha_I = 0.7$ )**

Figure 4 is the same plot as figure 3, except that  $\alpha_I$  is much larger (0.7). The same general shape holds with the following exceptions. First, the magnitude of distortion is much smaller. When  $\alpha_I = 0.7$ ,  $\delta$  never exceeds 0.15. When  $\alpha_I = 0.1$ ,  $\delta$  approached 0.8. This occurs because the maximum spread between  $\alpha_{II}$  and  $\alpha_I$  is smaller

when  $\alpha_L$  is large. Thus, there is less uncertainty, which suggests less need for distortion. The second difference between figures 3 and 4 is that in figure 4, increases in  $\delta$  are nearly linear in  $\alpha_H$ . In figure 3 the increases are initially very steep, but they quickly flatten. Finally, the skewness of the plot in figure 3 varies more than in figure 4. Again, this occurs because in figure 4, the range of the spread between  $\alpha_H$  and  $\alpha_L$  is 0.3, whereas the range in figure 3 is 0.9.

Recall that all of this distortion does not have an impact on either player's expected payoff. This is because, for any amount of distortion, the *total* expected theft and verification probabilities are the same for the non-sequential equilibrium and for this equilibrium. Thus, the two solutions are payoff equivalent ( $\frac{-2c}{\bar{\alpha}p}$ ). So, while the multiple location problem does not separate into independent single location problems, the strategic interaction between the controller and employee renders the information gathered through sequential verification useless in terms of improving the controller's expected payoff.

At this point I return to ADF to add some perspective. ADF proposed that their results applied to auditing settings. That is, because audits involve learning, sequential audit decisions do not decompose into independent single decisions. The results in this chapter are in many ways consistent with ADF's proposal. The overall problem does not decompose. In particular, the employee distorts his decisions from what would be optimal if the problem did decompose. However, this distortion has interesting effects.

First, it allows the controller to behave as if the problem did decompose. Second, it removes the controller's strategic advantage (the ability to verify sequentially, and use information learned about controls). Finally, the strategic interaction between the controller and employee ensures that neither party benefits from, or is harmed by the controller's advantage. This result is inconsistent with a single person setting. For instance, suppose the controller was merely searching for random errors due to an imperfect internal control system. In that setting, the controller would want to adopt a sequential strategy as long as learning about controls has the potential to affect future decisions. In this case, sequential verification would allow the controller to use the information about controls, and reduce losses from errors and verification costs.

## Chapter V

### Sequential Verification (Multiple Employees)

#### 5.1 Assumptions

In this section I consider the possibility of dividing responsibilities among employees, such that each location is in the custody of a single employee. It is easily verified that the solution from chapter IV holds for the multiple employee setting. Thus the controller can do at least as well with multiple employees.<sup>1</sup> The question is, can the controller do better? The answer is a qualified yes. In the multiple employee setting, there exists an equilibrium that does not exist in the single employee setting. The additional equilibrium has each employee choosing the same probability of theft. This equilibrium is preferred by the controller when penalty rates are sufficiently large. If penalty rates are “too small”, the controller prefers the multiple employee equivalent of Proposition 2, and the employees are indifferent. The new equilibrium also assumes that the employees act independently and do not coordinate their activities.<sup>2</sup>

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<sup>1</sup> This statement assumes that the equilibrium in Proposition 2 is unique. I conjecture that this is so, but it remains to be proved. This statement also assumes that there are no costs to division of responsibilities. This is unlikely to be true in practice. However, I assume this so that I may make direct comparisons between chapters 4 and 5.

<sup>2</sup> If the employees can coordinate their activities, the equilibrium in this section will not exist. However, the multiple employee equivalent of Proposition 2 still will exist.

In this new equilibrium, the controller does not revise his beliefs about whether he is at the high-theft location. He only revises his beliefs about the level of controls. I will present an example of this equilibrium. However, the conclusions hold in general.

In this example, since the employees both choose the same probability of theft, the controller only revises his beliefs about the expected level of controls. Following is an illustration of the controller's belief revision. The controller's belief about the expected level of controls, given the first location was investigated and a theft was detected is denoted as  $[\bar{\alpha} | A_1D_1]$ . This value is calculated according to Bayes' rule. It is:  $[\bar{\alpha} | A_1D_1] = \frac{k}{\bar{\alpha}}$ . Similarly,  $[\bar{\alpha} | A_1ND_1] = \frac{\bar{\alpha} - P T k}{1 - P T \bar{\alpha}}$ , where  $k = q \alpha_{H1}^2 + (1-q) \alpha_L^2$  and  $\bar{\alpha} = q \alpha_{H1} + (1-q) \alpha_L$ . Note that the controller's beliefs about  $[\bar{\alpha} | A_1ND_1]$  depend on  $T$  ( the probability that the employees will steal), which will be determined in equilibrium. I assume here that there exists an equilibrium with  $T_a = T_b = T$ . Proposition four shows that this is in fact true.

It is easily verified that for  $T^* > 0$ , the following ordering of beliefs obtains:  $[\bar{\alpha} | A_1D_1] > \bar{\alpha} > [\bar{\alpha} | A_1ND_1]$ . Thus, when the controller detects a theft, he revises his beliefs about the effectiveness of internal controls upward. Conversely, if no theft is detected, he revises downward.

The solution concept in this game is Bayes - Nash sequential equilibrium. The employees choose the probability of stealing from their locations, and the controller chooses the probability of verifying at Location 1, and the probability of verifying at

Location 2, given the outcome from the first location. His strategy for the second location must be sequentially rational and consistent with his beliefs about the effectiveness of the internal controls.

## **5.2 Solution**

I will now illustrate the additional equilibrium that obtains with multiple employees. This equilibrium assumes that the employees will not coordinate their activities. The controller prefers this equilibrium when penalties are sufficiently large. The equilibrium depends on the parameters. Proposition 3 summarizes.



### Proposition 3

When there is one employee per location, there exists an equilibrium in which each employee chooses the same probability of theft. The table below illustrates this equilibrium when  $V = 8$ ,  $C = 1$ ,  $P = 1$ ,  $\alpha_{H1} = .9$ ,  $\alpha_L = .7$ , and  $q = .5$  for both locations.

**Table 4**  
**Multiple Employee Equilibrium**

	$L < 1.2178$	$1.2178 \leq L \leq 1.2188$	$L > 1.2188$
Ta	0.1566	$\frac{1.2308(1.5-L)}{1+L}$	0.1559
Tb	0.1566	$\frac{1.2308(1.5-L)}{1+L}$	0.1559
A <sub>1</sub>	1	1	$\frac{2.2188}{1+L}$
[A <sub>2</sub>   A <sub>1</sub> D <sub>1</sub> ]	1	1	1
[A <sub>2</sub>   A <sub>1</sub> ND <sub>1</sub> ]	$\frac{1.2916(1.2178-L)}{1+L}$	0	0
[A <sub>2</sub>   NA <sub>1</sub> ]	1	*	0
EOL	2.5011	$\frac{19.6769(L-1.7018)(L+.0739)}{(1+L)^2}$	2.4957

\* 0 if  $L > 1.21837$ , 1 if  $L < 1.21837$

For this equilibrium to exist, the employees jointly must be willing to choose identical theft probabilities, given the controller's strategy. To verify that the employees will actually do this, consider the case where  $L > 1.2188$ . Each employee has two pure strategies {steal, don't steal}. Additionally, each employee's payoff depends on what

the other employee does. Taking the controller's strategy as given and looking at the employees' choices jointly we have:

**Table 5**  
**Employees' Best Responses**

		Employee B	
		Steal	Don't Steal
Employee A	Steal	-4.8690	0.8998 *
	Don't Steal	0 *	0

\* denotes a Nash equilibrium in pure strategies

In addition to the two pure strategy Nash equilibria, there exists a mixed strategy Nash equilibrium that has each employee stealing with probability 0.1559 ( $T_a$  and  $T_b$  from above). Furthermore, this is the only Nash equilibrium that is part of a Nash equilibrium in the overall game. However, the two pure strategy equilibria Pareto-dominate the mixed strategy equilibrium. If the employees can coordinate their activities, it seems reasonable to conclude that they will play one (or a combination) of the pure strategy equilibria, given the chance.

### 5.3 Analysis

The equilibrium in Proposition 3 has some interesting properties. One observation is that despite the fact that both locations are identical, learning enables the controller to investigate each with different probabilities. This result is consistent with Anderson and Young [1988], who also find that it can be optimal to treat identical locations differently.<sup>3</sup>

Note that the EOL is constant for “small penalties”. For “intermediate penalties”, the EOL decreases until the penalty rate becomes “large”. At this point, the EOL becomes constant again. Additionally, when the penalty is sufficiently small ( $L < 1.2180$ ), the controller prefers an equilibrium in which he ignores the information (multiple employee version of Proposition 2). When the penalty is sufficiently large ( $L > 1.2180$ ), the controller prefers the equilibrium from Proposition 3, so long as he is convinced that the employees will not coordinate their activities. This ordering raises an interesting question about the origin of the penalty parameter,  $L$ . In the model  $L$  is exogenous. However, in practice the penalty rate ( $L$ ) is determined by the firm or by perhaps the legal system. Clearly, if the firm could choose any  $L^* \in \mathbf{R}$ , it would choose  $L^* > 1.2180$ . Note that once  $L$  reaches 1.2188, EOL becomes constant. Thus there is no incentive to increase it further. If, however, the legal system determines  $L^*$ , or restricts the firm’s ability to set  $L^*$ , the firm may be worse off in this equilibrium than in the

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<sup>3</sup> Anderson and Young’s result however is not due to uncertainty about internal controls. Their result obtains because the internal auditor is operating under a budget constraint imposed by central management.



The most obvious result is that in the small penalty example, the expected organizational losses are greater in the Proposition 3 equilibrium than in the Proposition 2 equilibrium<sup>4</sup>. This occurs because there are costs implicit to using the information obtained through sequential verification in addition to benefits. A strategy that utilizes the information from sequential verification can be more costly to implement because I restrict admissible solutions to be sequentially rational.

To provide insight as to why the controller is worse off in Proposition 3 when penalties are low, I decompose expected organizational losses into two components, expected verification costs and expected losses from theft. First consider the large penalty example. In this case, expected verification costs are lower in Proposition 3 than in Proposition 2. Also in this case, expected losses from theft are lower in Proposition 3 than in Proposition 2.

Now, consider the small penalty example. In this example, expected verification costs again are lower in Proposition 3 than in Proposition 2. However, expected losses from theft are greater in Proposition 3 than in Proposition 2. Additionally, the magnitude of the increased losses from theft is greater than the magnitude of the savings in verification costs, making the controller worse off overall.

A natural question is, “why do penalty rates matter in Proposition 3. In both propositions, each player’s strategy must be a best response to the other player’s strategy. However, in Proposition 2, the employee’s strategy (which is independent of  $L$ ) makes

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<sup>4</sup> In the remainder of this section I will use the term, “Proposition 2” to refer to both the actual proposition, and its multiple employee equivalent.

the controller indifferent about verification in all situations. As a result, he has more degrees of freedom in choosing a strategy than in Proposition 3. In Proposition 3, the employees' strategies do not make the controller indifferent in all situations. As a result, the controller has less freedom in choosing a strategy. In particular, his actions in three of the four possible situations (first location, second location given detection, etc.) is fixed because of non-binding constraints.

In Proposition 3 with a small penalty, the only way for the controller to effectively deter theft is to always verify the first location, always verify the second location if he detected theft at the first location, and randomly verify the second location if no theft was detected at the first location. These actions jointly raise the probability of verification high enough to make the employees indifferent between stealing and not stealing. Given that the employees are indifferent, they will choose to steal just enough to keep the controller indifferent between verifying and not verifying at the second location when no theft was detected at the first location. But, note that in this situation, the controller has revised his beliefs about  $\alpha$  downward. Thus, relative to Proposition 2, the employees jointly steal more in order to compensate for the downward belief revision. In this situation, the employees are responding to the fact that the controller must act in a manner consistent with his beliefs, which have changed because of new information. The combination of increased theft, coupled with decreased verification (relative to the amount of verification in Proposition 2) implies that expected losses from theft will increase with sequential verification. This increase more than offsets cost savings from

decreased verification. As a result, the controller prefers the equilibria described in Propositions 1 and 2, which ensure a better expected payoff.

In summary, the magnitude of the penalty rate,  $L$ , plays an important role in determining how the controller wishes to carry out investigations. When penalty rates are relatively small, the controller prefers the single employee solution (Proposition 2), or its multiple employee equivalent, in which the information is useless. When penalty rates are relatively large, and the employees choose equal theft probabilities, the controller prefers the multiple employee solution, in which the information is useful.

## **Chapter VI**

### **Conclusion**

This paper has presented equilibrium arguments describing optimal strategies for detecting and preventing employee thefts in a firm with multiple assets that differ across three dimensions; value, cost of investigation, and detection risk. When verification is non-sequential, the controller's plan calls for investigation of high-risk locations with higher frequency than low-risk accounts. Additionally, all locations have a positive probability of being selected for verification. Detection risk also shows up in the controller's expected payoff. High-risk locations correspond to greater expected losses, and low risk locations correspond to smaller expected losses.

Comparative statics show that theft cannot be eliminated with excessively large penalties. Penalties must be sufficiently large to ensure that always stealing is not a dominant strategy for the employee. But once this threshold is reached, increasing the magnitude of the penalty has absolutely no impact on the amount of theft.

Finally, when the controller is permitted to adopt a sequential verification strategy, the multiple location problem fails to decompose into independent single location problems. However, there exists an equilibrium in which the employee destroys any incentive for the controller to use the information. The employee accomplishes this



by distorting the probability of theft at each location (relative to the probabilities that would be chosen if the problem did separate). As a result of this employee strategy, nobody benefits (or is harmed) from this decomposition.

There does exist an equilibrium with multiple employees in which the information about controls is useful. That is, the controller uses the information to modify his strategy, and his expected losses are reduced. In this equilibrium, the magnitude of the penalty imposed for detected thefts is important. This equilibrium is attractive only when penalties are sufficiently large. Otherwise, the controller prefers the “single employee” or non-sequential solutions.

There are some obvious limitations to this work. As a result, caution should be exercised in making policy statements based on this model. One limitation is that I have imposed a specific functional form on the penalty (proportional). Other penalty structures (i.e. a fixed penalty) may not produce the same results. A second limitation is that the sequential results are based on two locations. Whether the results hold for an arbitrary number of locations remains to be explored.

There are several other areas where extensions of this work might prove useful. The results from this model indicate that the presence of a strategic employee can limit the controller’s ability to use information obtained through verification procedures. It seems reasonable to consider the role that commitment might play in allowing the

controller to use the information obtained from verification. Also, it should be interesting to allow the locations to differ in the sequential models. This will make the sequencing of the investigation a strategic choice for the controller.

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## APPENDIX

### Proof of Proposition 1

If verification is non-sequential, the multiple location problem decomposes into independent single location problems. Thus, it is sufficient to examine a single representative location. The controller's objective function for location  $j$  is:

$$\Pi_j = A_j * T_j ((1 - \bar{\alpha} P_j)(-V_j) - C_j) + A_j(1 - T_j)(-C_j) + (1 - A_j)T_j(-V_j) + (1 - A_j)(1 - T_j)(0)$$

Where  $\bar{\alpha} = q \alpha_H + (1 - q) \alpha_L$ . This represents the expected level of controls.

Simplifying, we get:

$$\Pi_j = A_j (T_j P_j V_j \bar{\alpha} - C_j) - T_j V_j \tag{1}$$

Recall that  $P_j V_j \alpha_L > C_j$ , which implies that  $P_j V_j \bar{\alpha} - C_j > 0$ .

The controller's objective is linear in  $A_j$ . Thus, if  $T_j > \frac{C_j}{\bar{\alpha} P_j V_j}$ , the controller's best response is to choose

$A_j = 1$ . If  $T_j < \frac{C_j}{\bar{\alpha} P_j V_j}$ , the controller's best response is to choose  $A_j = 0$ . If  $T_j = \frac{C_j}{\bar{\alpha} P_j V_j}$ , then the

controller is indifferent as to the choice of  $A_j$ .

The employee has the following objective function:

$$U_j = A_j * T_j ((\bar{\alpha} P_j)(-V_j L) + (1 - \bar{\alpha} P_j)(V_j)) + A_j(1 - T_j)(0) + (1 - A_j)T_j(V_j) + (1 - A_j)(1 - T_j)(0)$$

Simplifying, we get:

$$U_j = T_j V_j (1 - A_j P_j \bar{\alpha} (1 + L)) \tag{2}$$

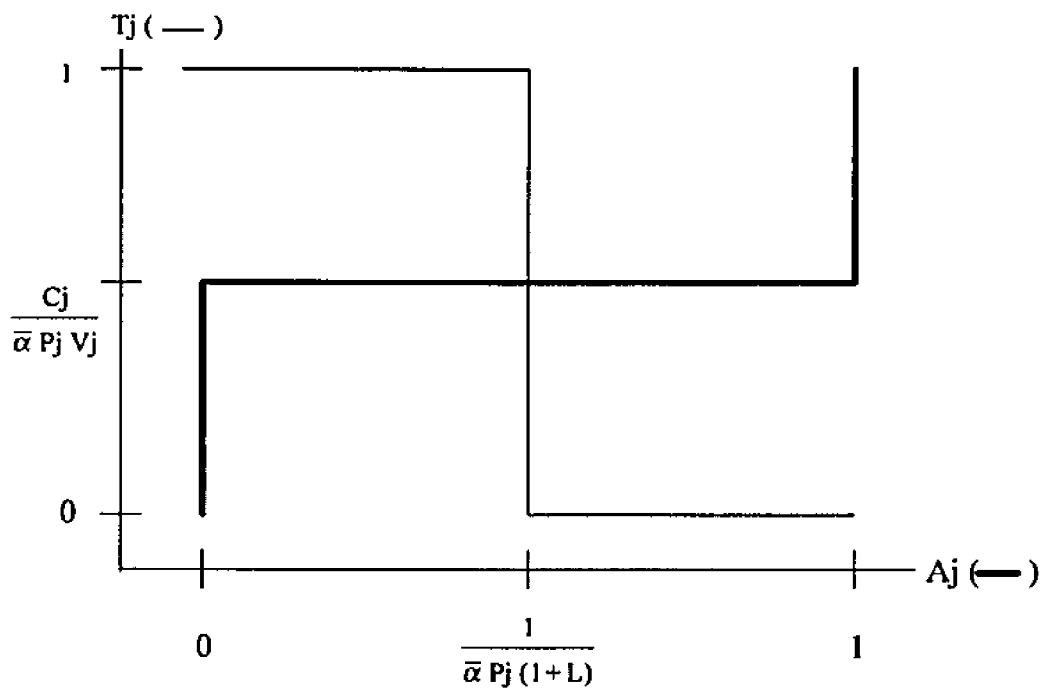
Recall that  $P_j \bar{\alpha} (1 + L) > P_j \alpha_L (1 + L) > 1$ .

The employee's objective is linear in  $T_j$ . Thus, if  $A_j > \frac{1}{\bar{\alpha} P_j (1 + L)}$ , the employee's best response is to

choose  $T_j = 0$ . If  $A_j < \frac{1}{\bar{\alpha} P_j (1 + L)}$ , the employee's best response is to choose  $T_j = 1$ . If  $A_j = \frac{1}{\bar{\alpha} P_j (1 + L)}$ ,

then the employee is indifferent as to the choice of  $T_j$ .

To determine the Nash equilibrium solution to this game, consider the following graph:



**Figure 5**  
Best Response Functions

The intersection of the best response functions on this graph shows that the only strategy combination that are mutual best responses is  $A_j = \frac{1}{\bar{\alpha} P_j (1+L)}$  and  $T_j = \frac{C_j}{\bar{\alpha} P_j V_j}$ . This pair is the Nash equilibrium solution to the non-sequential model.



### Proof of Lemma 1

In This proof, I show that there is no equilibrium in which a single employee chooses  $T_a = T_b$ .

The employee's first order conditions from proposition 2 are:

$$\frac{\partial U}{\partial T_a} : v/2 \{ 2 - \bar{\alpha} p (1+L) (A_1 + [A_2 | A_1 ND_1] A_1 + [A_2 | NA_1] (1-A_1)) + 2T_b A_1 \frac{k}{\bar{\alpha}} p ([A_2 | A_1 D_1] - [A_2 | A_1 ND_1]) \} = 0 \quad (ca)$$

$$\frac{\partial U}{\partial T_b} : v/2 \{ 2 - \bar{\alpha} p (1+L) (A_1 + [A_2 | A_1 ND_1] A_1 + [A_2 | NA_1] (1-A_1)) + 2T_a A_1 \frac{k}{\bar{\alpha}} p ([A_2 | A_1 D_1] - [A_2 | A_1 ND_1]) \} = 0 \quad (cb)$$

Assume the controller believes  $T_a = T_b \equiv T$ .

The controller's objective function is written as:

$$\begin{aligned} \Pi = & A_1 (T \bar{\alpha} P V - C) - T V + \\ & A_1 D_1 \{ [A_2 | A_1 D_1] (T [\bar{\alpha} | A_1 D_1] P V - C) - T V \} + \\ & A_1 ND_1 \{ [A_2 | A_1 ND_1] (T [\bar{\alpha} | A_1 ND_1] P V - C) - T V \} + \\ & NA_1 \{ [A_2 | NA_1] (T \bar{\alpha} P V - C) - T V \} \end{aligned} \quad (c1)$$

$$\frac{\partial \Pi}{\partial A_1} : (T \bar{\alpha} v p - c) (1 - [A_2 | NA_1] + [A_2 | A_1 ND_1]) + (k p^2 T^2 v - \bar{\alpha} c p T) ([A_2 | A_1 D_1] - [A_2 | A_1 ND_1]) \quad (c2)$$

To prove that there is no equilibrium with  $T_a = T_b \equiv T$ , I use backward induction.

At the second location, the controller will face one of the following problems:

$$\begin{aligned} \text{Given } A_1 D_1: & \quad \text{Max: } [A_2 | A_1 D_1] (T [\bar{\alpha} | A_1 D_1] P V - C) - T V. \\ & = \text{Max: } [A_2 | A_1 D_1] (T \frac{k}{\bar{\alpha}} P V - C) - T V. \end{aligned} \quad (c3)$$

$$\begin{aligned} \text{Given } A_1 ND_1: & \quad \text{Max: } [A_2 | A_1 ND_1] (T [\bar{\alpha} | A_1 ND_1] P V - C) - T V. \\ & = \text{Max: } [A_2 | A_1 ND_1] (T (\frac{\bar{\alpha} - P T k}{1 - P T \bar{\alpha}}) P V - C) - T V. \end{aligned} \quad (c4)$$

$$\begin{aligned} \text{Given } NA_1: & \quad \text{Max: } [A_2 | NA_1] (T [\bar{\alpha} | NA_1] P V - C) - T V. \\ & = \text{Max: } [A_2 | NA_1] (T \bar{\alpha} P V - C) - T V. \end{aligned} \quad (c5)$$



The following table presents the optimal verification probabilities at the second location, given the outcomes at the first location and given a particular level of theft:

**Table 7**  
Optimal Verification Probabilities - Single Employee

Case		$[A_2   A_1 D_1]$	$[A_2   A_1 N D_1]$	$[A_2   N A_1]$
1	$T > \frac{\alpha(c+v) \cdot \sqrt{\alpha^2(c+v)^2 - 4ckv}}{2kpv}$	1	1	1
2	$T = \frac{\alpha(c+v) \cdot \sqrt{\alpha^2(c+v)^2 - 4ckv}}{2kpv}$	1	arbitrary	1
3	$\frac{\alpha(c+v) \cdot \sqrt{\alpha^2(c+v)^2 - 4ckv}}{2kpv} > T > \frac{c}{\bar{\alpha}pv}$	1	0	1
4	$T = \frac{c}{\bar{\alpha}pv}$	1	0	arbitrary
5	$\frac{c}{\bar{\alpha}pv} > T > \frac{\alpha(c-v) + \sqrt{\alpha^2(c-v)^2 + 4ckv}}{2kpv}$	1	0	0
6	$T = \frac{\alpha(c-v) + \sqrt{\alpha^2(c-v)^2 + 4ckv}}{2kpv}$	1	0	0
7	$\frac{\alpha(c-v) + \sqrt{\alpha^2(c-v)^2 + 4ckv}}{2kpv} > T > \frac{\bar{\alpha}c}{kpv}$	1	0	0
8	$T = \frac{\bar{\alpha}c}{kpv}$	arbitrary	0	0
9	$T < \frac{\bar{\alpha}c}{kpv}$	0	0	0

Case 1.

Assume  $T > \frac{\bar{\alpha}(c+v) \cdot \sqrt{\alpha^2(c+v)^2 - 4ckv}}{2kpv}$ . I claim there is no equilibrium for this case. Proof: For the assumed magnitude of  $T$ , it follows that  $[A_2 | A_1 D_1] = 1$ ,  $[A_2 | A_1 N D_1] = 1$ , and  $[A_2 | N A_1] = 1$ .

Substituting these values into (c2) indicates that (c2) is positive when  $T > \frac{\bar{\alpha}(c+v) \cdot \sqrt{\alpha^2(c+v)^2 - 4ckv}}{2kpv}$ .

So  $A_1 = 1$ . Now substitute  $[A_2 | A_1 D_1] = [A_2 | A_1 N D_1] = [A_2 | N A_1] = A_1 = 1$  into that (ca) and (cb). Since  $\bar{\alpha}p(1+L) > 1$  by assumption, it follows that (ca) and (cb) are negative. Therefore  $T = 0$ , which

contradicts the initial assumption that  $T > \frac{\bar{\alpha}(c+v) - \sqrt{\bar{\alpha}^2(c+v)^2 - 4ckv}}{2kp v}$ . Thus, there cannot be an

equilibrium with  $T > \frac{\bar{\alpha}(c+v) - \sqrt{\bar{\alpha}^2(c+v)^2 - 4ckv}}{2kp v}$ . ■

Case 2.

Assume  $T = \frac{\bar{\alpha}(c+v) - \sqrt{\bar{\alpha}^2(c+v)^2 - 4ckv}}{2kp v}$ . It follows that  $[A_2 | A_1 D_1] = 1$ ,  $[A_2 | N A_1] = 1$ , and  $[A_2 | A_1 N D_1]$  arbitrary. Substituting these values into (c2) makes (c2) positive, so  $A_1 = 1$ .

Given that the employee is choosing  $T = \frac{\bar{\alpha}(c+v) - \sqrt{\bar{\alpha}^2(c+v)^2 - 4ckv}}{2kp v}$  at one location, the following audit probability will make him willing to randomize at the other location:

$$[A_2 | A_1 N D_1] = \frac{p(1+L)\beta - 2v(1-\bar{\alpha}p(1+L))}{p(1+L)\beta} \text{ where } \beta = \bar{\alpha}c - \sqrt{\bar{\alpha}^2(c+v)^2 - 4ckv}.$$

Assume this value is between 0 and 1. If not, we do not have an equilibrium and the proof of case 2 is done.

So, given  $T = \frac{\bar{\alpha}(c+v) - \sqrt{\bar{\alpha}^2(c+v)^2 - 4ckv}}{2kp v}$ , the audit probabilities specified above constitute a best

response. I now take these probabilities as given and show that  $T = \frac{\bar{\alpha}(c+v) - \sqrt{\bar{\alpha}^2(c+v)^2 - 4ckv}}{2kp v}$  is NOT a best response for the employee.

After substituting the controller's audit probabilities into the employee's objective function, it takes the following form:

$$\begin{aligned} U &= T_a \{ \phi - \gamma T_b \} + T_b \{ \phi - \gamma T_a \} \\ &= \phi(T_a + T_b) - 2\gamma T_a T_b \end{aligned} \quad (c6)$$

where  $\phi$  and  $\gamma$  were determined by the controller's strategy. Inspection of (ca) and (cb) indicates that for the audit probabilities under consideration, we have  $\gamma > 0$ .

Taking first order conditions produces:

$$\frac{\partial U}{\partial T_a} : \phi - 2\gamma T_b \quad (c7)$$

$$\frac{\partial U}{\partial T_b} : \phi - 2\gamma T_a \quad (c8)$$

I claim that there is never a setting in these circumstances for which  $T_a = T_b = \text{interior}$  is an optimal solution. There are three cases to consider:

- a)  $\phi \leq 0$ . In this case (c7) and (c8) are negative for all possible  $T_a$  and  $T_b$ . So,  $T_a = T_b = 0$  maximizes (c6).
- b)  $\phi \geq 2\gamma$ . In this case (c7) and (c8) are non-negative for all possible  $T_a$  and  $T_b$ . So,  $T_a = T_b = 1$  maximizes (c6).
- c)  $0 < \phi < 2\gamma$  In this case, setting (c7) and (c8) = 0 and solving for  $T_a$  and  $T_b$  produces:  $T_a = T_b = \frac{\phi}{2\gamma}$ .

Substituting this into (c6) yields an expected payoff of  $\frac{\phi^2}{2\gamma}$ . However, consider the following strategy:  $T_a = 0, T_b = 1$ . This strategy satisfies (c7) and (c8), and produces an expected payoff of  $\phi$ . Since  $\phi < 2\gamma$ , it follows that  $\phi > \frac{\phi^2}{2\gamma}$ .

So, there is no equilibrium with  $T_a = T_b = \frac{\bar{\alpha}(c+v) - \sqrt{\bar{\alpha}^2(c+v)^2 - 4ckv}}{2kpv}$ . ■

The proofs of the non-existence of equilibria for the other cases are similar, and hence omitted.

### Proof of Proposition 2

In this proposition, I assume the employee will choose different theft probabilities for each location. I later verify that this is an equilibrium strategy.

I assume that the employee arbitrarily designates a location as location a (b), and steals from that location with probability  $T_a$  ( $T_b$ ). The controller investigates, not knowing which location is location a or which is location b.

In this setting, the controller makes inferences about two things. He makes inferences about the expected level of controls, and, he makes inferences about which location he is at (a or b).

Consider the case where he has detected a theft at the first location.

Define the following probabilities (conditional on detection at the first location):

$$\frac{T_b \alpha_{Hq}}{(T_a + T_b) \bar{\alpha}} \equiv \text{probability that the second location is location A, and controls are effective } (\alpha_{H1}).$$

$$\frac{T_b \alpha_{L(1-q)}}{(T_a + T_b) \bar{\alpha}} \equiv \text{probability that the second location is location A, and controls are ineffective } (\alpha_{L1}).$$

$$\frac{T_a \alpha_{Hq}}{(T_a + T_b) \bar{\alpha}} \equiv \text{probability that the second location is location B, and controls are effective } (\alpha_{H1}).$$

$$\frac{T_a \alpha_{L(1-q)}}{(T_a + T_b) \bar{\alpha}} \equiv \text{probability that the second location is location B, and controls are ineffective } (\alpha_{L1}).$$

The controller's second location problem, given detection at the first location is weighted by these probabilities: After some simplification, the problem looks like:

$$srd1 \equiv [A_2 | A_1 D_1] \left\{ \left( \frac{2T_a T_b}{T_a + T_b} \right) \left( \frac{k}{\bar{\alpha}} \right) p v - c \right\} - \left( \frac{2T_a T_b}{T_a + T_b} \right) v \quad (srd1)$$

The controller makes a similar set of inferences, given that he did not detect a theft at the first location. These inferences result in the following problem at the second location, given no detection:

$$srd1 \equiv [A_2 | A_1 N D_1] \left\{ \left( \frac{(T_a + T_b) \bar{\alpha} - 2 T_a T_b p k}{2 - \bar{\alpha} p (T_a + T_b)} \right) p v - c \right\} - \left( \frac{(T_a + T_b) - 2 T_a T_b p \bar{\alpha}}{2 - \bar{\alpha} p (T_a + T_b)} \right) v \quad (srd1)$$

Finally, If the controller did not investigate the first location, his second location problem looks like:

$$sma1 \equiv [A_2 | N A_1] \left\{ \left( \frac{T_a + T_b}{2} \right) \bar{\alpha} p v - c \right\} - \frac{T_a + T_b}{2} v \quad (sma1)$$

At the first location, the controller must consider how his actions will impact the second location problem. Accordingly his first location problem looks like:

$$\text{Max}_{A_1} : \left\{ [A_1] \left\{ \left( \frac{T_a + T_b}{2} \right) \bar{\alpha} p v - c \right\} - \frac{T_a + T_b}{2} v + \left\{ A_1 \left( \frac{T_a + T_b}{2} \right) \bar{\alpha} p \{srd\} + A_1 \left( 1 - \left( \frac{T_a + T_b}{2} \right) \bar{\alpha} p \right) \{smd\} + (1 - A_1) \{sma\} \right\} \right\} \quad (3.1)$$

Note that in this problem I use backwards induction. Thus the second location choices are taken as given.

Now I turn to the employee's problem. In this section he has access to both locations. I assume he chooses the probability of stealing from each location simultaneously. Also, when choosing his strategy he will anticipate revisions his beliefs about the effectiveness of controls. Specifically:

$\frac{k}{\bar{\alpha}}$  = the expected level of controls, given a theft occurred at the other location, and given no detection.  
 $\frac{\bar{\alpha} - kp}{1 - \bar{\alpha}p}$  = the expected level of controls, given a theft occurred at the other location, and given detection.

The employee's problem looks like:

$$\text{Max}_{T_a, T_b} U = 0.5v T_a \{ (1 - A_1 \bar{\alpha} p(1+L)) + \quad (1)$$

$$T_b \left\{ \begin{array}{l} A_1 \bar{\alpha} p (1 - [A_2 | A_1 D_1] (\frac{k}{\bar{\alpha}}) p (1+L)) + \\ A_1 (1 - \bar{\alpha} p) (1 - [A_2 | A_1 ND_1] (\frac{\bar{\alpha} - kp}{1 - \bar{\alpha}p}) p (1+L)) + \\ (1 - A_1) (1 - [A_2 | NA_1] \bar{\alpha} p (1+L)) \end{array} \right\} + (1 - T_b) \left\{ \begin{array}{l} A_1 (1 - [A_2 | A_1 ND_1] \bar{\alpha} p (1+L)) + \\ (1 - A_1) (1 - [A_2 | NA_1] \bar{\alpha} p (1+L)) \end{array} \right\} \quad (2)$$

$$0.5v T_b \{ (1 - A_1 \bar{\alpha} p(1+L)) + \quad (3)$$

$$T_a \left\{ \begin{array}{l} A_1 \bar{\alpha} p (1 - [A_2 | A_1 D_1] (\frac{k}{\bar{\alpha}}) p (1+L)) + \\ A_1 (1 - \bar{\alpha} p) (1 - [A_2 | A_1 ND_1] (\frac{\bar{\alpha} - kp}{1 - \bar{\alpha}p}) p (1+L)) + \\ (1 - A_1) (1 - [A_2 | NA_1] \bar{\alpha} p (1+L)) \end{array} \right\} + (1 - T_a) \left\{ \begin{array}{l} A_1 (1 - [A_2 | A_1 ND_1] \bar{\alpha} p (1+L)) + \\ (1 - A_1) (1 - [A_2 | NA_1] \bar{\alpha} p (1+L)) \end{array} \right\} \quad (4)$$

There are four lines to this objective function. Line 1(3) is the employee's payoff at location a (b) if it is the first location selected by the controller. Line 2 (4) is the employees payoff at location a (b) if it is the second location selected.

This problem reduces to:

$$0.5v T_a \{ (1 - A_1 \bar{\alpha} p(1+L)) + \left\{ \begin{array}{l} A_1 T_b \bar{\alpha} p (1 - [A_2 | A_1 D_1] (\frac{k}{\bar{\alpha}}) p (1+L)) + \\ A_1 (1 - \bar{\alpha} p T_b) (1 - [A_2 | A_1 ND_1] (\frac{\bar{\alpha} - kp T_b}{1 - \bar{\alpha} p T_b}) p (1+L)) + \\ (1 - A_1) (1 - [A_2 | NA_1] \bar{\alpha} p (1+L)) \end{array} \right\} \quad +$$

$$0.5v T_b \left\{ (1 - A_1 \bar{\alpha} p(1+L)) + \left[ \begin{array}{l} A_1 T_a \bar{\alpha} p(1 - [A_2 | A_1 D_1] (\frac{k}{\bar{\alpha}}) p(1+L)) + \\ A_1 (1 - \bar{\alpha} p T_a) (1 - [A_2 | A_1 N D_1] (\frac{\bar{\alpha} - k p T_a}{1 - \bar{\alpha} p T_a}) p(1+L)) + \\ (1 - A_1) (1 - [A_2 | N A_1] \bar{\alpha} p(1+L)) \end{array} \right] \right\} \quad (3.2)$$

The employee will be willing to randomize at location A if:

$$\frac{\partial U}{\partial T_a} : v/2 \{ 2 - p(1+L) \{ \bar{\alpha} (A_1 + [A_2 | A_1 N D_1] A_1 + [A_2 | N A_1] (1 - A_1)) + 2 T_b A_1 k p ([A_2 | A_1 D_1] - [A_2 | A_1 N D_1]) \} \} = 0 \quad (3.3a)$$

He will be willing to randomize at location B if:

$$\frac{\partial U}{\partial T_b} : v/2 \{ 2 - p(1+L) \{ \bar{\alpha} (A_1 + [A_2 | A_1 N D_1] A_1 + [A_2 | N A_1] (1 - A_1)) + 2 T_a A_1 k p ([A_2 | A_1 D_1] - [A_2 | A_1 N D_1]) \} \} = 0 \quad (3.3b)$$

I will now prove the existence of an equilibrium where the employee chooses different interior probabilities for theft at each location.

Claim: When the employee chooses:  $T_a = \frac{c}{\bar{\alpha} p v} \left\{ 1 + \sqrt{\frac{k - \bar{\alpha}^2}{k}} \right\}$  and  $T_b = \frac{c}{\bar{\alpha} p v} \left\{ 1 - \sqrt{\frac{k - \bar{\alpha}^2}{k}} \right\}$ , the controller is indifferent between investigating and not investigating at each location:

Proof: Substitute  $T_a = \frac{c}{\bar{\alpha} p v} \left\{ 1 + \sqrt{\frac{k - \bar{\alpha}^2}{k}} \right\}$  and  $T_b = \frac{c}{\bar{\alpha} p v} \left\{ 1 - \sqrt{\frac{k - \bar{\alpha}^2}{k}} \right\}$  into expressions

(srd1), (srnd1) and (srmal). Each expression is equal to a constant, which means there is no marginal benefit or cost to verification at the second location. Thus the controller is indifferent as to the probability of verification at the second location. Additionally, substituting the above values of  $T_a$  and  $T_b$  into the controller's first location objective (3.1), leaves this expression equal to  $\frac{-2c}{\bar{\alpha} p}$ . This expression is a constant,

independent of  $A_1$ . This means that the controller is indifferent between verifying and not verifying at the first location no matter what he will find at the first location and no matter what he will do at the second location. So, given the employee's strategy, any response is a best response for the controller.

Claim: When the controller chooses

$$a) [A_2 | N A_1] = \frac{2}{\bar{\alpha} p (1 + L)} \text{ and } A_1 = 0$$

or

$$b) [A_2 | A_1 D_1] = [A_2 | A_1 N D_1] \equiv [A_2 | A_1], \text{ and}$$

$$A_1 + \{ A_1 [A_2 | A_1] + (1 - A_1) [A_2 | N A_1] \} = \frac{2}{\bar{\alpha} p (1 + L)}$$

the employee is willing to randomize at each location, and his choice at each location is independent of his

choice at the other location. Thus any choice of  $T_a$  and  $T_b$  ( including  $T_a = \frac{c}{\bar{\alpha} p v} \left\{ 1 + \sqrt{\frac{k - \bar{\alpha}^2}{k}} \right\}$ ,

and  $T_b = \frac{c}{\bar{\alpha} p v} \left\{ 1 - \sqrt{\frac{k - \bar{\alpha}^2}{k}} \right\}$  ), is a best response.

**Proof:** If the employee is to be willing to randomize at each location, it must be that both

(3.3a) = 0 and (3.3b) = 0. If (3.3a) = 0 and (3.3b) = 0, then (3.3a)-(3.3b) = 0.

But (3.3a)-(3.3b) =  $A_1 \{ [A_2 | A_1 D_1] - [A_2 | A_1 N D_1] \} (T_a - T_b) (v \bar{\alpha}^2 p^2 (1+L))$ . Since  $T_a \neq T_b$ , the only way that (3.3a)=(3.3b) is if  $A_1=0$ , or if  $[A_2 | A_1 D_1] = [A_2 | A_1 N D_1]$ .

It remains to find conditions under which both (3.3a) and (3.3b) are equal to zero.

**Part a).**

Suppose  $A_1 = 0$ . Then (3.3a) = (3.3b) =  $v/2 \{ 2 - \bar{\alpha} p (1+L) [A_2 | N A_1] \}$ .

This expression is equal to zero when  $[A_2 | N A_1] = \frac{2}{\bar{\alpha} p (1+L)}$ .

**Part b).**

Suppose  $[A_2 | A_1 D_1] = [A_2 | A_1 N D_1]$ . Then

(3.3a) = (3.3b) =  $v/2 \{ 2 - \bar{\alpha} p (1+L) (A_1 + [A_2 | A_1 N D_1] A_1 + [A_2 | N A_1] (1-A_1)) \}$ .

This expression is equal to zero when  $A_1 + \{ A_1 [A_2 | A_1] + (1 - A_1) [A_2 | N A_1] \} = \frac{2}{\bar{\alpha} p (1+L)}$ .

Thus, given the controller's strategy (either Part a or Part b), any choice of  $T_a$  and  $T_b$

( including  $T_a = \frac{c}{\bar{\alpha} p v} \left\{ 1 + \sqrt{\frac{k - \bar{\alpha}^2}{k}} \right\}$ , and  $T_b = \frac{c}{\bar{\alpha} p v} \left\{ 1 - \sqrt{\frac{k - \bar{\alpha}^2}{k}} \right\}$  ), is a best response. ■

### Proof of Proposition 3

In this proposition, I consider the case where  $V=8$ ,  $c=1$ ,  $p=1$ ,  $a_H = 0.9$ ,  $a_L = 0.7$ , and  $q = 0.5$  for both locations. Since both locations are identical, the controller chooses a particular location with probability  $1/2$ . I assume here that the employees will choose equal theft probabilities. Later, I will verify this.

The solution concept is Bayes - Nash sequential equilibrium. Since the controller gathers information sequentially, he will revise his beliefs about the expected value of  $\alpha$ . The controller revises his beliefs in accordance with Bayes' Theorem.

Assume  $T_a = T_b \equiv T$ .

Let the controller's belief about the expected level of controls, given the first location was investigated and a theft was detected be denoted as  $[\bar{\alpha} | A_1 D_1]$ . This value is calculated according to Bayes' rule:

$$[\bar{\alpha} | A_1 D_1] \equiv \frac{k}{\bar{\alpha}} = 0.8125$$

$$\text{Similarly, } [\bar{\alpha} | A_1 N D_1] \equiv \frac{\bar{\alpha} - P T k}{1 - P T \bar{\alpha}} = \frac{0.8 - 0.65 T}{1 - 0.8 T}$$

Where  $k = q \alpha_H^2 + (1-q) \alpha_L^2 = 0.65$ , and that  $\bar{\alpha} = q \alpha_H + (1-q) \alpha_L = 0.8$ . Simple calculations indicate that for any  $\alpha_H > \alpha_L$  the following is true:  $\bar{\alpha} > k > \bar{\alpha}^2$ .

Additionally, define the following expected probabilities:

Expected probability of investigating the first location and detecting theft:  $A_1 D_1 \equiv A_1 T \bar{\alpha} P = 0.8 A_1 T$

Expected probability of investigating the first location and not detecting theft:

$$A_1 N D_1 \equiv A_1 (1-T \bar{\alpha} P) = A_1 (1-0.8 T)$$

Expected probability of not investigating the first location:  $N A_1 \equiv 1 - A_1$

The employees choose their theft strategies not knowing which location will be the first or which location will be the second. I arbitrarily label one employee "employee a" and the other "employee b". Borrowing from proposition 2, their objective functions are:

$$U_a: \quad 0.5(8) T_a \left\{ (1 - A_1 0.8(1+L)) + \frac{A_1 T_b 0.8(1 - [A_2 | A_1 D_1](0.8125)(1+L)) + A_1 (1-0.8T_b)(1 - [A_2 | A_1 N D_1](\frac{0.8-0.65T_b}{1-0.8T_b})(1+L))}{(1 - A_1)(1 - [A_2 | N A_1] 0.8(1+L))} \right\} \quad (4.1)$$

$$U_b: \quad 0.5(8) T_b \left\{ (1 - A_1 0.8(1+L)) + \frac{A_1 T_a 0.8(1 - [A_2 | A_1 D_1](0.8125)(1+L)) + A_1 (1-0.8T_a)(1 - [A_2 | A_1 N D_1](\frac{0.8-0.65T_a}{1-0.8T_a})(1+L))}{(1 - A_1)(1 - [A_2 | N A_1] 0.8(1+L))} \right\} \quad (4.2)$$

The controller learns about the control environment after investigating the first location. Accordingly he updates his beliefs about the expected level of controls. The controller's objective function (ex-ante) is written as:



$$\begin{aligned}
\Pi = & A_1(8(0.8)T - 1) - 8T + \\
& (0.8 A_1 T) \{ [A_2 | A_1 D_1] (8(0.8125)T - 1) - 8T \} + \\
& (A_1 (1-0.8T)) \{ [A_2 | A_1 ND_1] (8(\frac{0.8 - 0.65T}{1 - 0.8T})T - 1) - 8T \} + \\
& (1-A_1) \{ [A_2 | NA_1] (8(0.8)T - 1) - 8T \}
\end{aligned} \tag{4.3}$$

Taking first order conditions yields:

$$\frac{\partial U_a}{\partial T_a} : 4\{2 - (1+L)(0.8(A_1 + [A_2 | A_1 ND_1] A_1 + [A_2 | NA_1](1-A_1)) + T_b A_1 0.65 ([A_2 | A_1 D_1] - [A_2 | A_1 ND_1])) \} \tag{4.4a}$$

$$\frac{\partial U_b}{\partial T_b} : 4\{2 - (1+L)(0.8(A_1 + [A_2 | A_1 ND_1] A_1 + [A_2 | NA_1](1-A_1)) + T_a A_1 0.65 ([A_2 | A_1 D_1] - [A_2 | A_1 ND_1])) \} \tag{4.4b}$$

$$\frac{\partial \Pi}{\partial A_1} : (8(0.8)T - 1)(1 - [A_2 | NA_1] + [A_2 | A_1 ND_1]) + (8(0.65)T^2 - 0.8T)([A_2 | A_1 D_1] - [A_2 | A_1 ND_1]) \tag{4.5}$$

I will use backwards induction to determine all existing equilibria. I will show the controller's optimal second location strategies for any given level of theft. I will then determine an optimal first location strategy and theft level.

At the second location, the controller will face one of the following problems:

$$\begin{aligned}
\text{Given } A_1 D_1: & \quad \text{Max: } [A_2 | A_1 D_1] (T [\bar{\alpha} | A_1 D_1] P V - C) - T V. \\
& = \text{Max: } [A_2 | A_1 D_1] (T (0.8125)(8) - 1) - 8 T.
\end{aligned} \tag{4.6}$$

$$\begin{aligned}
\text{Given } A_1 ND_1: & \quad \text{Max: } [A_2 | A_1 ND_1] (T [\bar{\alpha} | A_1 ND_1] P V - C) - T V. \\
& = \text{Max: } [A_2 | A_1 ND_1] (8(\frac{0.8 - 0.65T}{1 - 0.8T})T - 1) - 8 T.
\end{aligned} \tag{4.7}$$

$$\begin{aligned}
\text{Given } NA_1: & \quad \text{Max: } [A_2 | NA_1] (T [\bar{\alpha} | NA_1] P V - C) - T V. \\
& = \text{Max: } [A_2 | NA_1] (8(0.8)T - 1) - 8 T.
\end{aligned} \tag{4.8}$$

The following table presents the optimal verification probabilities at the second location, given the outcomes at the first location and given a particular level of theft:

**Table 8**  
**Optimal Verification Probabilities: Multiple Employees**

Case		$[A_2   A_1 D_1]$	$[A_2   A_1 N D_1]$	$[A_2   N A_1]$
1	$T > 0.15660$	1	1	1
2	$T = 0.15660$	1	arbitrary	1
3	$0.15660 > T > 0.15625$	1	0	1
4	$T = 0.15625$	1	0	arbitrary
5	$0.15625 > T > 0.15598$	1	0	0
6	$T = 0.15598$	1	0	0
7	$0.15598 > T > 0.15385$	1	0	0
8	$T = 0.15385$	arbitrary	0	0
9	$T < 0.15385$	0	0	0

To prove the existence of equilibria, I take these second location strategies as given, and determine values for  $T$  and  $A_1$  that maximize each players' objective function, that are best responses given the other player's strategy, and that are consistent with assumptions about second location behavior.

Case 1.

Assume  $T > 0.1566$ . I claim there are no equilibria for this case. Proof: For the assumed magnitude of  $T$ , it follows that  $[A_2 | A_1 D_1] = 1$ ,  $[A_2 | A_1 N D_1] = 1$ , and  $[A_2 | N A_1] = 1$ . Substituting these values into (4.5) indicates that (4.5) is positive when  $T > 0.1566$ . So  $A_1 = 1$ . Now substitute  $[A_2 | A_1 D_1] = [A_2 | A_1 N D_1] = [A_2 | N A_1] = A_1 = 1$  into (4.4a) and (4.4b). Since  $\bar{\alpha} p(1+L) > 1$  by assumption, it follows that (4.4a) and (4.4b) are negative. Therefore

$T = 0$ , which contradicts the initial assumption that  $T > 0.1566$ . Thus, there cannot be an equilibrium with  $T > 0.1566$ . ■

Case 2.

Assume  $T = 0.1566$ . It follows that  $[A_2 | A_1 D_1] = 1$ ,  $[A_2 | N A_1] = 1$ , and  $[A_2 | A_1 N D_1]$  arbitrary. Substituting these values into (4.5) makes (4.5) positive, so  $A_1 = 1$ .

So, given  $T = 0.1566$ , the controller will choose  $[A_2 | A_1 D_1] = 1$ ,  $[A_2 | N A_1] = 1$ , and  $[A_2 | A_1 N D_1]$  arbitrary.

Thus he is willing to choose  $[A_2 | A_1 N D_1] = \frac{1.29157(1.21781 - L)}{1 + L}$ . This expression is interior when  $L < 1.21781$ .

Conversely, given  $[A_2 | A_1 D_1] = 1$ ,  $[A_2 | N A_1] = 1$ ,  $A_1 = 1$  and  $[A_2 | A_1 N D_1] = \frac{1.29157(1.21781 - L)}{1 + L}$ , it remains to verify that each employee will choose  $T = 0.1566$ .

To verify this, I take the controller's strategy as given, examine what the two employees jointly will do. Recall from (4.4a) and (4.4b) that each employee's payoff depends on the other employee's strategy. Each

employee makes a binary choice (steal, not steal). I will show that there exists a Nash equilibrium where each employee plays a mixed strategy, choosing to steal with probability  $T = 0.1566$ .

Recall that  $L < 1.21781$  in this setting. Also, it was initially assumed that  $L > 0.4286$  (because  $\alpha_L p(1+L) > 1$ ). Consider the following normal form representation:

Table 9  
Employees' Best Responses

		Employee a	
		Steal	Don't Steal
Emp. b	Steal	$\frac{5.9581(0.2109 - 0.6326L - 0.8434 L^2)}{1 + L}$	$0.9330 (L - 0.2499)$ *
	Don't Steal	$\frac{5.9581(0.2109 - 0.6326L - 0.8434 L^2)}{1 + L}$	0
		0 *	$0.9330 (L - 0.2499)$
			0

It is easily verified that  $T_a = T_b = 0.1566$  is a mixed strategy Nash equilibrium. Furthermore, this is the only Nash equilibrium that is also part of a Nash equilibrium in the overall game. However, the (\*) represents two other Nash equilibria, that the employees may jointly play. These equilibria are not part of a Nash equilibrium in the overall game. Furthermore, these two solutions Pareto-dominate the mixed strategy solution.

To summarize, given the controller's strategy, one best response has each employee choosing  $T = 0.1566$ . Conversely, given that each employee is choosing  $T = 0.1566$ , the controller is willing to choose  $[A_2]$

$[A_1|D_1] = 1$ ,  $[A_2|NA_1] = 1$ ,  $A_1 = 1$  and  $[A_2|A_1ND_1] = \frac{1.29157(1.21781 - L)}{1 + L}$ . Furthermore, these choices

are sequentially rational, given the controller's beliefs about the expected level of controls. Thus we have a sequential equilibrium.

The proof of the other equilibria are similar, and hence omitted.

